NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ALTERNATIVE MODELS FOR CALCULATION OF ELEVATION ANGLES AND RAY TRANSIT TIMES FOR RAY TRACING OF HYDROPHONIC TRACKING DATA

by

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Alternative Models for Calculation of Elevation Angles and Ray Transit Times for Ray Tracing of Hydrophonic Tracking Data

by

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This thesis treats the problem of determining the elevation angle needed to initialize an underwater sound ray tracing algorithm used to locate the position of a target vehicle. At regularly spaced time intervals the vehicle pings a synchronized sound signal which is received by a (short base line) sonar array containing four hydrophones positioned at four of the corners of a cube. The wavefront direction angles are determined from the arrival times at the four hydrophones.

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I. INTRODUCTION AND BACKGROUND

A. BACKGROUND

Suppose that an underwater acoustical source emits a signal at a known time. Suppose that the times of arrival of that signal at the hydrophones on a three dimensional sensing array are also known. Finally suppose that the speed of sound in water is modelled as being homogeneous over time and horizontal displacement, varying only with depth. Then the angle of elevation (A), and the time (T) of the arrival of the signal at the acoustic center of the hydrophone array can be determined. If the relationship between the speed of sound and the depth under water is known exactly, then the angle A and the time T can be used to trace the signal trajectory back over its ray path (called ray tracing) to determine the original position of the acoustical source [Ref. 1]. However, full realization of this method in actual hydrophonic tracking is prevented by two primary sources of inaccuracies in the process. The first and probably greatest problem is that the speed of sound profile can be approximated at only a few locations, usually at great cost to achieve even moderate accuracy. In addition the profile is certain to flucuate over time and location. The second problem, confounding the first, is that there may be innacuracies, of unknown size, in the time data values recorded by the sensing array.

B. PURPOSE

As noted in [Ref. 1] the procedure of determining a sound source position by ray tracing is very sensitive to even small errors in the angle of elevation or speed of

sound at the sensing array. Of course ray tracing is also highly dependent on the accurate determination of the the transit time from source to array. The purpose of this study is to develop an appropriate model of the timing data observed in the hydrophonic tracking problem. The objective of the desired model is to estimate the error in the time values, and produce improved estimates of the initial angle and ray transit time, so as to reduce the effect of those errors on the ray tracing procedure.

In pursuit of these objectives this thesis first reviews the currently used models, which are called the NAVY and NAVY_A methods. Then four alternative models are developed, called the L.S., L.S.C., M.L.P. and M.L.S. methods. Finally the performance of all six models are evaluated and compared using simulation studies. The comparisons are made under the idealized conditions of a known linear speed of sound versus depth relationship.

C. TRACKING RANGE CONFIGURATION

The tracking range which supplied data for this study consists of several separate three dimensional hydrophone arrays sitting on the sea bottom. They are layed out in a rough grid, each array being approximately 7500 feet from the next, so that the sound source being tracked is never more than about 5000 feet from the nearest array. The arrays are at depths of roughly 1000 to 1300 feet.

Each array (see figure 1.1) has four independent hydrophones defining an orthogonal coordinate system. The phones are referred to as the x,y,z, and c hydrophones, and are on four adjacent vertices of a cube (see figure 1.2) with sides of length D (usually 30 feet). The arrays are linked to shore based computers by electronic cable. The origin of the local coordinate system of each array is at the acoustic

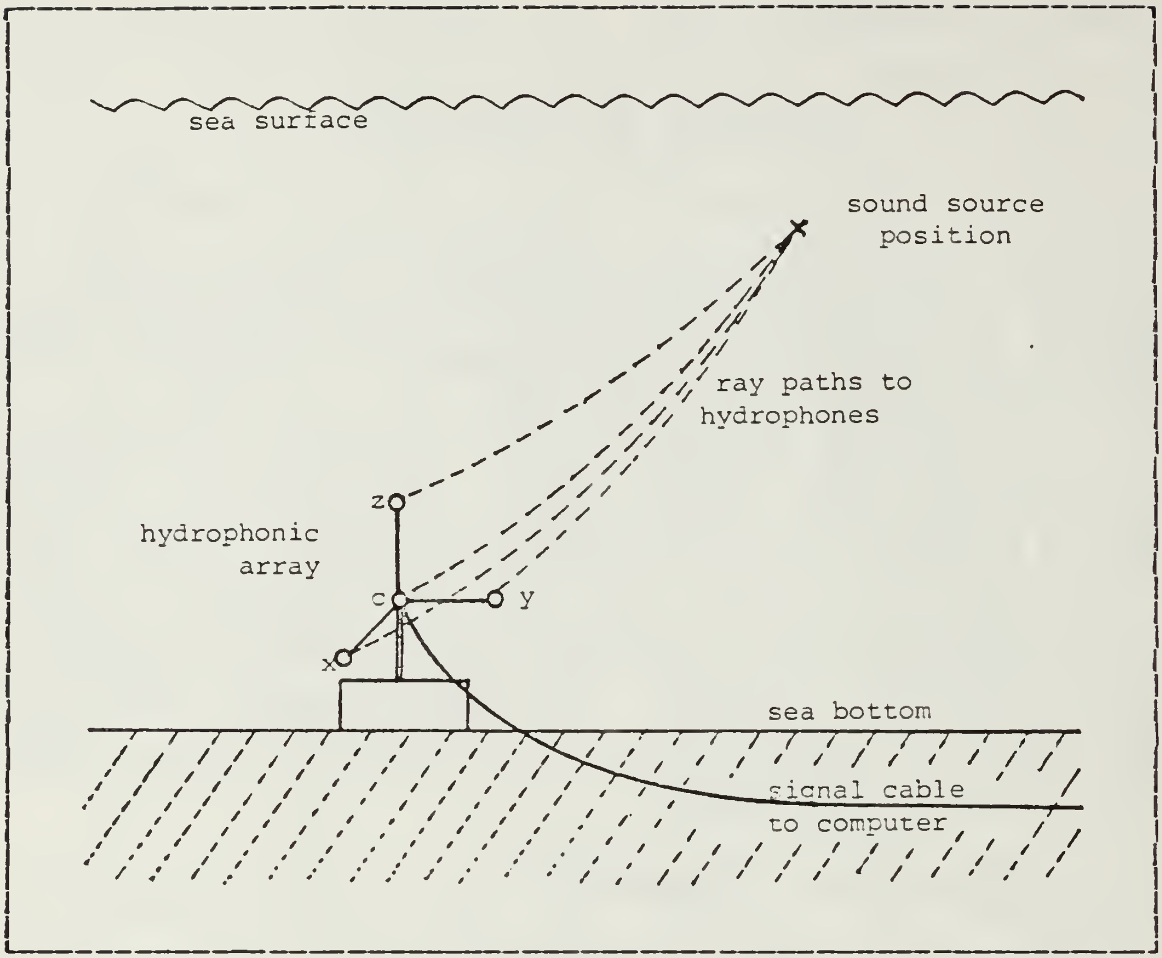


Figure 1.1 Acoustic Signal Detection by Three Dimensional Hydrophonic Array.

center of the array, which is the center of the cube defined by the four hydrophones. Therefore the hydrophones are in the positions

$$
\begin{array}{ll}
x: & (\;D\,,\,-D\,,\,-D\;)\;/\;2 \\
y: & (\;-D\,,\;D\,,\,-D\;)\;/\;2 \\
z: & (\;-D\,,\,-D\,,\;D\;)\;/\;2 \\
c: & (\;-D\,,\,-D\,,\,-D\;)\;/\;2
\end{array}
$$

in terms of the array's local coordinate system.

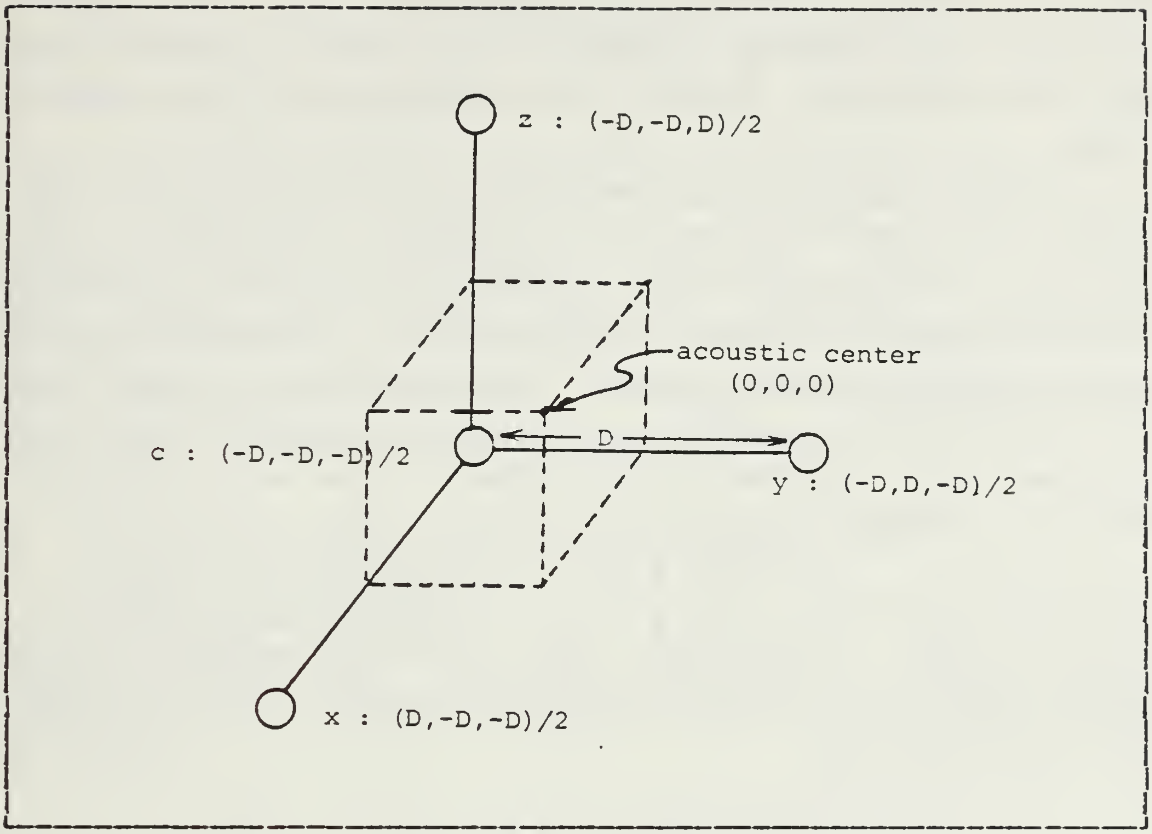


Figure 1.2 Geometry of Hydrophonic Arrays.

The sound source is equipped with a clock synchronized with the shore based computers and emits a signal at specified intervals. The times of receipt of those signals at the four hydrophones are recorded, and the corresponding travel times are calculated by subtraction of the signal emission time.

D. APPARENT POSITIONS

The first step in estimating the position of a sound source is to do so under the assumption that the sound wave travelled its entire trajectory through water which had a constant speed of sound. The result, called the apparent position, is obviously erroneous, but is then corrected by

the ray tracing procedure (a description of which follows later). The constant velocity value used is usually the estimated speed of sound V at the depth of the sensing array.

The constant velocity assumption implies that the wavefront is an expanding sphere. Therefore the apparent position is calculated using simple spherical equations involving squared distance calculations. Specifically, if Tx is the travel time recorded for the x-phone, then V•Tx is the distance between the apparent position and the x-phone. This distance is also equal to the usual geometric distance between the two positions

$$(X , Y , Z) \qquad \text{(apparent position)}$$

$$\text{and} \quad (D ,-D ,-D) / 2 \qquad \text{(x hydrophone position).}$$

Equating these two squared distances, and equating their counterparts for the other three hydrophones, the equations (1.1) are obtained.

$$\begin{aligned} (X - D/2)^2 + (Y + D/2)^2 + (Z + D/2)^2 &= V^2 T_x^2 \qquad (1.1) \\ (X + D/2)^2 + (Y - D/2)^2 + (Z + D/2)^2 &= V^2 T_y^2 \\ (X + D/2)^2 + (Y + D/2)^2 + (Z - D/2)^2 &= V^2 T_z^2 \\ (X + D/2)^2 + (Y + D/2)^2 + (Z + D/2)^2 &= V^2 T_c^2 \end{aligned}$$

Assuming that the times Tx, Ty, Tz, and Tc are known, (1.1) is a system of four equations in three unknowns X, Y, and Z. This overdetermined system will, in general, have no exact solution. In fact, even if the time values were exactly correct, the system would still have no exact solution. This is because the equations correspond to the straight line ray paths due to the constant velocity assumption, whereas the time values correspond to the true ray paths which are not straight due to the actual variation of velocity of sound along the ray path. This is a subtle, but very important point.

To throw out one of the equations arbitrarily, so as to reduce the system to three equations in three unknowns, is to throw away information from one of the hydrophones. The psuedo solution currently utilized is to subtract the fourth equation from each of the first three, yielding a system of three equations in three unknowns which allows an exact solution involving information from all four hydrophones. However that solution will not, in general, satisfy any of the original four equations, and is only one of many reaonable ways to choose an approximate solution.

E. INITIAL ELEVATION ANGLE AND RAY TRANSIT TIME

Assuming that a solution (Xa,Ya,Za) has been determined for the apparent position, then the initial angle of elevation is just the angle of elevation of that solution, given by (1.2).

$$A_1 = \arcsin\left(Z_a \Big/ \sqrt{X_a^2 + Y_a^2 + Z_a^2}\right) \quad . \qquad (1.2)$$

The objective is to find an apparent position (Xa,Ya,Za) such that (1.2) computes an angle which approximates the physically correct elevation angle as closely as possible. The solution (Xa,Ya,Za) and the times Tx, Ty, Tz and Tc can then be used to determine an appropriate value for T, which is the 'ray transit time', or time of arrival of the sound wave at the acoustic center of the sensing array. The currently employed method uses the proportional relationship of equation (1.3), where R and Rc are the ranges from the apparent position to the acoustic center and to the c hydrophone respectively, as in (1.4).

$$\frac{R_c}{T_c} = V_1 = \frac{R}{T} \quad \left(\text{or} \quad T = \frac{T_c\ R}{R_c} \right) \qquad (1.3)$$

$$R = \sqrt{X_a^2 + Y_a^2 + Z_a^2}$$

$$R_c = \sqrt{(X_a+D/2)^2 + (Y_a+D/2)^2 + (Z_a+D/2)^2} \qquad (1.4)$$

F. RAY TRACING

Whichever method is selected to produce the apparent position (Xa,Ya,Za), it is transformed into the estimate of the true sound source position (X,Y,Z) by the procedure of ray tracing. When there is velocity layering in the water, the ray path is no longer a straight line. This is treated using repeated applications of Snell's Law [Ref. 2 p.131], starting with the layer of water in which the array sits, and backtracking upwards through successive velocity layers, until the estimated ray transit time T is consumed.

The layering effect is artificially induced by the limitation that the speed of sound can be estimated at only a finite number of depths, the result of which is commonly called the water column. For example, at the tracking range studied the speed of sound is measured every 25 feet, starting at the depth of 12.5 feet. Hence, for example, the third layer from 50 to 75 feet deep is assumed to have a constant speed of sound equal to that measured at 62.5 feet.

The first layer processed [Ref. 3 p.4] is the partial layer lying between the array and the deepest layer boundary that is shallower than the array, with thickness Z1 (see figure 1.3).

The incremental slant range in the first layer is S1 given by (1.5), where A1 is the initial elevation angle estimate.

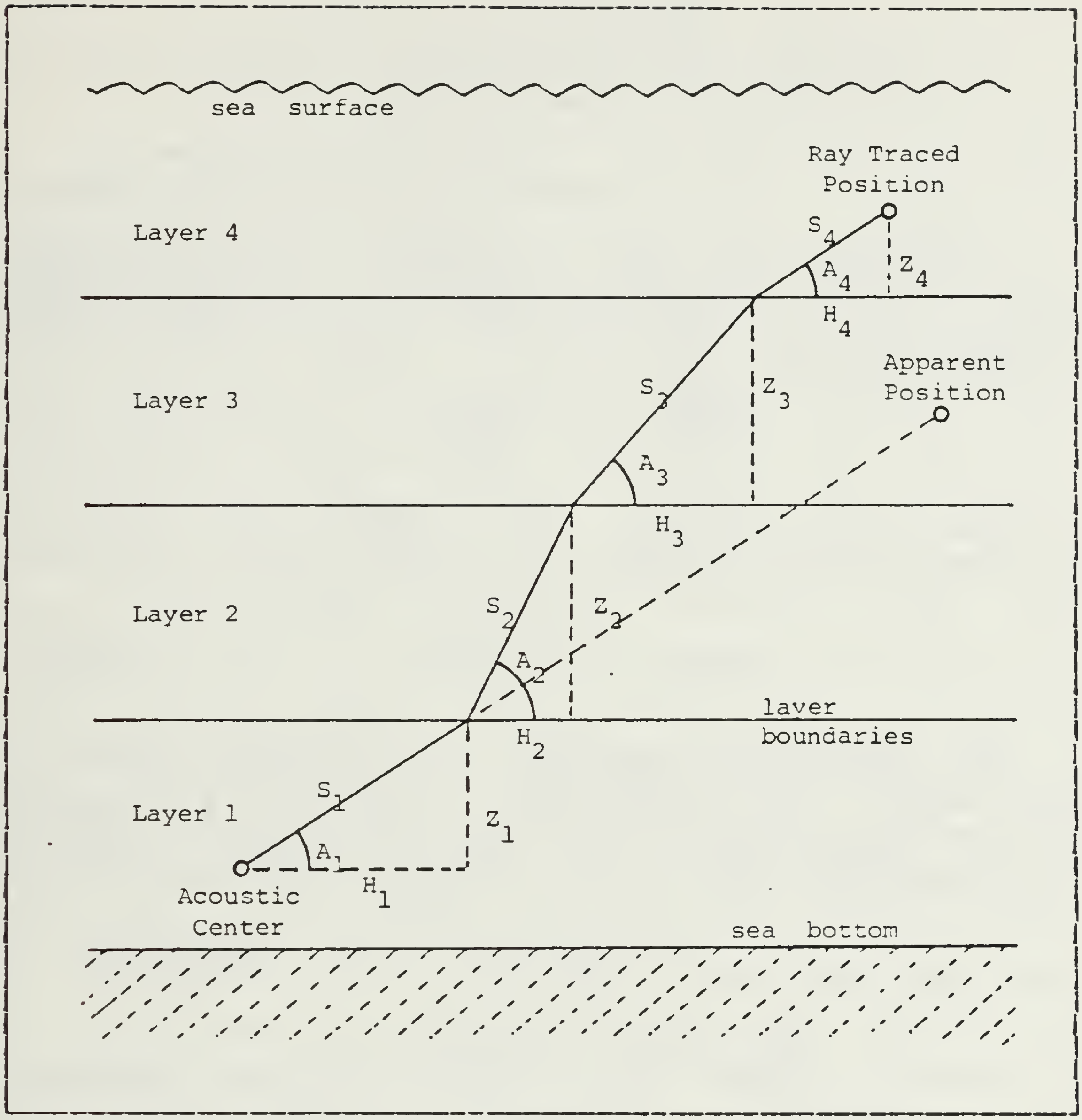


Figure 1.3 Ray Tracing an Apparent Position.

$$S_1 = Z_1 / \sin(A_1) \tag{1.5}$$

The incremental travel time in the first layer is T1 given by (1.6), where V1 is the velocity estimated for the layer in which the array is situated.

$$T_1 = S_1 / V_1 \tag{1.6}$$

The incremental horizontal distance travelled by the ray in the first layer is H1 given by (1.7).

$$H_1 = S_1 \cos(A_1) \tag{1.7}$$

To determine the angle of elevation in the next layer, Snell's Law (1.8) is applied, where V2 is the speed of sound estimated for that layer.

$$\frac{\cos(A_1)}{V_1} = \frac{\cos(A_2)}{V_2} \tag{1.8}$$

When (1.8) is solved for the cosine of the angle of entry into the next layer, (1.9) is obtained.

$$\cos(A_2) = \frac{V_2 \cos(A_1)}{V_1} \tag{1.9}$$

The procedure of computing the incremental values of slant range, time and horizontal distance are now repeated for the second layer. The overall procedure is repeated upwards through layers 2,...,n , where n is the first layer in which the sum of the incremental travel times exceeds the total ray transit time, as in (1.10).

$$\left(T_1 + T_2 + \ldots + T_n\right) > T \tag{1.10}$$

In the last, uppermost layer the values Tn, Sn, Hn, and Zn must be adjusted to compensate for overshooting the total time T. The values Hi and Zi (i=1,...,n) are then accumulated to get (1.11).

$$H = \sum_{i=1}^{n} H_i \qquad Z = \sum_{i=1}^{n} Z_i \tag{1.11}$$

Now the raytraced position estimate is given by (1.12).

$$X = (X_a H)\Big/\sqrt{X_a^2 + Y_a^2}$$

$$Y = (Y_a H)\Big/\sqrt{X_a^2 + Y_a^2} \qquad (1.12)$$

$$Z = Z$$

The sensing array is usually not aligned with the coordinate system of the overall tracking range. Therefore the apparent position, which is in terms of the local array coordinates, must be changed by a suitable geometric transformation prior to ray tracing so as to account for the angle of tilt at which the array sits on the sea bottom. After ray tracing, the position estimate must be again transformed to account for rotation of the array about its Z axis away from a position which is aligned with the range coordinate axes. Finally a simple translation must be applied to reference the position estimate to the range coordinate system origin vice the acoustic center of the array. The end result is a position in terms of the overall range coordinate system. These transformations are not given here because they are used after the estimation of the initial angle and time, and hence do not affect the accuracy of those estimates. See [Ref. 3] for further details.

G. DISCUSSION

If the velocity versus depth relationship is smooth and estimated accurately, then the ray tracing procedure is surprisingly robust with respect to the thickness of the layers. For example, if velocity is linear versus depth, and is known exactly, then the exact hydrophone times, ray transit time and initial angles can be computed [Ref. 4] for any given sound source position. Then the ray tracing procedure, with layers as thick as 25 feet and targets as far away as 3000 feet, still estimates positions to within inches of each of the true coordinate values. This seems to indicate that errors resulting from position estimation are

not due to the approximation by layers, except possibly when there are radical changes in the velocity pattern within single layers such as frequently occur in layers near the water surface. Rather such errors apparently are due mostly to inaccuracies either in the estimation of the speed of sound profile itself, or in the initial angle and transit time estimates. This study will focus on those errors which are involved in the time values observed at the hydrophones, and attempt to produce methods of initial angle and transit time estimation which reduce the effects of those errors on the overall position estimation procedure.

II. CURRENTLY EMPLOYED METHODS

A. BASIC CONCEPTS

The method currently used for estimation of an initial angle and ray path transit time focuses on the four spherical equations (1.1) given in Chapter I. As previously discussed, these equations have no exact solution because:

1. they form an overdetermined system of four equations in three unknowns;
2. the time values recorded at the hydrophones may be innacurate due to unknown sources of error in the observation process; and
3. even if the time values were exact, they correspond to a nonconstant velocity profile, and so will not be be correct for the constant velocity geometry (spherical wavefront) assumed by the equations.

As previously mentioned, the psuedo solution chosen by the current method is to subtract the fourth spherical equation from each of the first three, and solve the resulting system of three equations in three unknowns. This method has the beneficial quality that information is retained from all four hydrophones, whereas to just drop the fourth equation (or any one of the equations) without the initial subtraction would cause complete loss of the information from the data recorded by one of the hydrophones. However it is important to realize that the solution thus obtained does not actually satisfy any of the original four equations.

It should be noted that the development of this solution in [Ref. 3] is done entirely from a geometrical point of view, and does not mention the system of four spherical

equations. The text of [Ref. 3] does not draw attention to the fact that the solution developed is just one of many plausible choices, none of which will satisfy all four spherical constraints. Therefore the solution chosen is treated as though it were the exact solution, only subject to errors in the observed hydrophone time values. However, even with exactly correct time values, this currently employed method will not yield the true elevation angle and ray transit time. This is due to the assumption of a constant velocity vice the true nonconstant velocity profile. This conflict introduces an automatic bias in the initial angle and time estimates currently used for ray tracing.

B. COMPUTATIONS

To simplify notation, let (X,Y,Z) be the coordinates of the apparent position which was formerly denoted (Xa,Ya,Za). Then when the current solution is applied, the first step is to subtract the fourth equation from each of the other three, which produces the equations (2.1).

$$\begin{aligned} (X - D/2)^2 - (X + D/2)^2 &= V^2 (T_c^2 - T_x^2) \\ (Y - D/2)^2 - (Y + D/2)^2 &= V^2 (T_c^2 - T_y^2) \\ (Z - D/2)^2 - (Z + D/2)^2 &= V^2 (T_c^2 - T_z^2) \end{aligned} \qquad (2.1)$$

The solution to these are easily obtained, as in (2.2).

$$\begin{aligned} X &= V^2 (T_c^2 - T_x^2) / 2D \\ Y &= V^2 (T_c^2 - T_y^2) / 2D \\ Z &= V^2 (T_c^2 - T_z^2) / 2D \end{aligned} \qquad (2.2)$$

Then the initial elevation angle estimate is (2.3).

$$A = \arcsin\left(Z \Big/ \sqrt{X^2 + Y^2 + Z^2} \right) \qquad (2.3)$$

The ray path transit time to the acoustic center is (2.4), where R and Rc are as defined in Chapter I by (1.4).

$$T = T_c \, R \,/\, R_c \tag{2.4}$$

This method of determining the apparent position shall hereafter be referred to as the 'Navy Method', or 'NAVY' for short.

C. ADJUSTMENTS TO THE ORIGINAL SOLUTION

Experience has shown that the NAVY method produces an apparent position estimate which usually can be improved by an adjustment which is described in this section.

The cosine of the angle between the i-th axis and the straight line from the origin out to the apparent position is called the i-th direction cosine Ci. It is a fact of geometry that the sum of the squares of the three direction cosines must equal unity. Therefore the method is now adjusted to reflect that constraint.

The direction cosines used are the angles made by the ray path at the c hydrophone. Therefore the (X,Y,Z) coordinates calculated by the original method are first translated to coordinates referenced to the c-phone as the temporary origin, as in (2.5).

$$X_c = X + D/2 \qquad Y_c = Y + D/2 \qquad Z_c = Z + D/2 \tag{2.5}$$

Therefore the three direction cosines are given by (2.6).

$$C_x = X_c \,/\, V\,T_c \qquad C_y = Y_c \,/\, V\,T_c \qquad C_z = Z_c \,/\, V\,T_c \tag{2.6}$$

The denominators in (2.6) are all V•Tc because that is the range from the apparent position to the c hydrophone, as estimated by the time from the c hydrophone. Ideally these

cosines should add to unity when squared. Therefore if DCC is the 'direction cosines correction' factor defined by (2.7), then DCC should be close to one.

$$DCC = \sqrt{C_x^2 + C_y^2 + C_z^2} \tag{2.7}$$

Deviation of DCC from unity is interpreted as an indication of receiver timing errors, array malformation or invalid data at one or more of the hydrophones [Ref. 3 p.C-3]. Currently if DCC lies outside the interval (0.98,1.02), the data is discarded as being excessively full of error. The direction cosines of the remaining acceptable data points are rescaled using (2.8) to assure satisfaction of the direction cosines constraint.

$$C'_x = C_x / DCC \qquad C'_y = C_y / DCC \qquad C'_z = C_z / DCC \tag{2.8}$$

A corrected set of new coordinates are computed by (2.9), still being referenced to the c hydrophone.

$$X_c = V\,T_c\,C'_x \qquad Y_c = V\,T_c\,C'_y \qquad Z_c = V\,T_c\,C'_z \tag{2.9}$$

These are then translated by (2.10) to coordinates referenced to the acoustic center.

$$X = X_c - D/2 \qquad Y = Y_c - D/2 \qquad Z = Z_c - D/2 \tag{2.10}$$

This adjusted method of determining the apparent position shall hereafter be referred to as the 'Navy Adjusted Method', or 'NAVY_A' for short.

D. DISCUSSION

When a sound source is within the detection range of more than one sensing array, each array produces timing data. The data from each array can be processed to produce

a position estimate. Ideally these multiple estimates of position will be in reasonably close agreement. However experience with actual tracking data has shown that this is not the case in many of the multiple detection opportunities. This tendency toward disagreement between multiple estimates of the same position is commonly called the crossover, or crosstalk, problem. This problem often occurs when the sound source is moving away from the tracking domain of one array into the tracking domain of another. This study focuses on improvement of the initial angle and time estimates, which hopefully will help alleviate the crossover problem.

The current choice of a 'best' compromise solution appears to be based on reasons of simplifying geometry and calculations. These are worthwhile objectives, but do not in themselves reflect the need to estimate accurately the initial elevation angle and ray transit time. Since there exist physically correct values for both the angle and the time, those values will produce the exact position after ray tracing, provided that the velocity profile is known exactly. The desire then is to estimate these true values as accurately as possible.

The question at this point is whether or not the direction cosines adjustment causes the estimated apparent position to be closer to the true apparent position. Experience has indicated that it does [Ref. 3 p.C-7]. However the effect of the adjustment can be interpreted in terms of the original four spherical equations (1.1). The rescaling of the direction cosines given by (2.6), so as to assure that their squares add to unity, is equivalent to rescaling the quantities in (2.5) so as to assure that their squares add to $(V \cdot Tc)^2$. That is exactly the constraint stated by the fourth spherical equation of (1.1). So the effect of the adjustment is to require that the fourth equation,

concerning the data at hydrophone c, is exactly satisfied. This requirement will, in general, assure that the other three equations are not satisfied. Since experience shows that the adjustment often improves the solution, this seems to imply that the fourth equation is somehow more important than the other three. Or it may just be that exact satisfaction of one of the equations usually assures a reasonably good compromise solution.

To summarize, the NAVY method provides a useable apparent position suitable as input for ray tracing. But the direction cosines adjustment used in the NAVY_A method, for reasons not understood at this time, appears to improve that position as indicated by test results. Those results are supported by the results of this thesis (see Chapter V). However, as will be demonstrated by the example considered in the next section, the DCC correction factor of the NAVY_A method has an effect which must be something more than just the smoothing of timing errors.

E. A COMPUTATIONAL EXAMPLE

For the purposes of illustration and comparison, suppose that a 30 foot sensing array is at a depth of 1300 feet, that the coordinate system origin is at the array acoustic center, and that the array arms are parallel to the coordinate axes. This implies that the four hydrophones are in the positions:

$$x : (15, -15, -15)$$
$$y : (-15, 15, -15)$$
$$z : (-15, -15, 15)$$
$$c : (-15, -15, -15) \quad .$$

If there is a sound source known to be in position

$$(1000, 3000, 900)$$

then the depth of that source is 1300 - 900 = 400 feet.

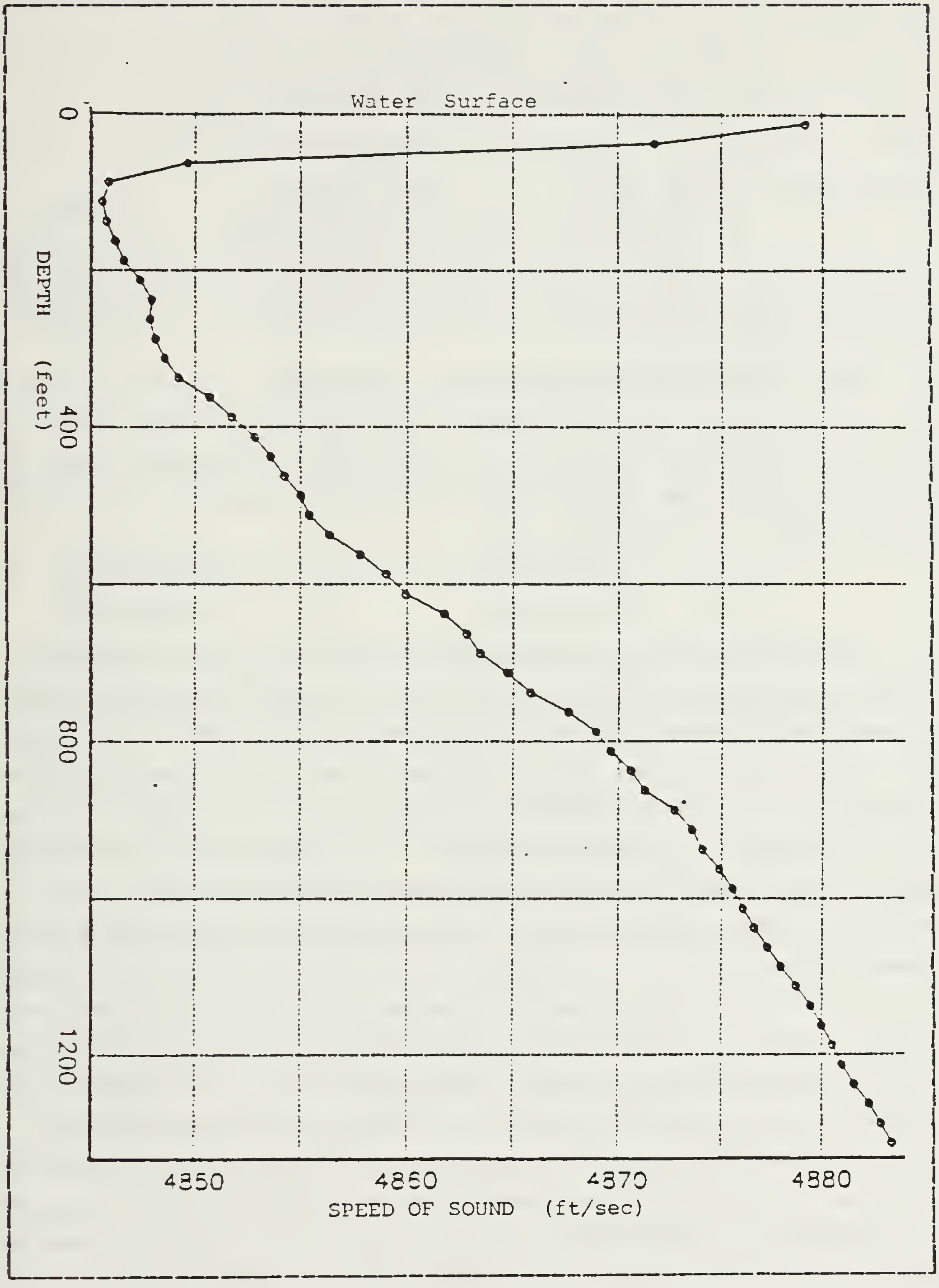


Figure 2.1 Sample Depth Versus Velocity Profile.

Figure 2.1 shows the estimated sound velocity profile which was estimated for the data used in the course of this thesis. As can be seen, the profile is primarily linear at depths greater than 100 feet. The profile at depths greater than 100 feet is reasonably approximated by the linear relationship

$$V = 4840.7 + 0.03314 \bullet \text{DEPTH} .$$

Therefore suppose that in this example problem the velocity profile is known exactly, and is given by the above linear relationship.

Under these circumstances, with known linear velocity profile, and known sound source location, the exact times of arrival of the sound wave at the four hydrophones can be computed using the methods set forth in Appendix A. Those exact times (in seconds) are:

$$T_x : 0.6779686893 \qquad T_y : 0.6742257788$$
$$T_z : 0.6782243197 \qquad T_c : 0.6798324156 .$$

The corresponding exact values for the initial elevation angle, ray transit time and resulting apparent position are also directly computable. Those computations will hereafter be be known as the EXACT method, and will produce the correct true position after ray tracing. The results of the EXACT method are given in Table I, along with the corresponding apparent position estimates produced when the two methods, NAVY and NAVY_A, are applied to the (errorless) time values.

At first glance the differences in Table I might seem rather small. However it is important to recall that these are produced under the ideal conditions of a very smooth and exactly known velocity profile. These idealizations are far from the realities of a nonlinear velocity profile which is estimated by a procedure involving errors which are unknown and probably significant. Such realities might well cause the differences in Table I to be significantly larger. The

TABLE I

Single Example Comparison of NAVY and NAVY_A Methods

Method	Transit Time T (secs)		Elev. Angle A (degs)
NAVY	0.67526969		15.26459
NAVY_A	0.67527027		15.26461
EXACT	0.67527043		15.27002
	Apparent	Position	Estimate
NAVY	(1005.957 ,	3017.874 ,	868.143)
NAVY_A	(1006.087 ,	3018.259 ,	868.255)
EXACT	(1005.966 ,	3017.899 ,	868.555)

nature and size of those differences remain difficult to determine until more is known about the the velocity profile estimation errors and their effect on the position estimating process. In any event even the small differences in Table I might be magnified during the ray tracing process under the conditions of a realistic velocity profile.

The differences illustrate the very important point that <u>the direction cosines adjustment causes changes in the estimates even when the time data is free of all error</u>. Hence the deviation from 1.0 of the correction factor DCC is not just due to array malfunction, receiver timing error or other sources of non-valid data as previously assumed.

In the example above, the NAVY_A method produced a slightly better time and angle value than the NAVY method. However, in this same example the NAVY_A method produced an apparent position estimate which is slightly farther away from the EXACT answer than the position estimated by the

NAVY method. This is only one example, and its results should not be generalized. However it illustrates the point that apparently the true effect of the adjustment may not be well understood.

Furthermore, the adjustment seems to place heavy emphasis on the time value recorded at hydrophone c. Therefore the effect of the adjustment may well depend largely on the accuracy of that one particular data value, which is a relatively unbalanced dependence in the presence of data errors.

III. PLANAR WAVEFRONT MODELS

A. THE PLANAR WAVEFRONT ASSUMPTION

When a sound wave travels in constant velocity water, it expands in the shape of a sphere. If the velocity profile is variable instead, but is reasonably well behaved versus depth, then the expanding wave is a smooth distortion of a spherical surface. In either case, if the wave has travelled a long distance when it arrives at a hydrophonic array, then that small piece of the wavefront which passes through the 30 foot cube spanned by the array may be approximated reasonably by a flat planar surface. This approximation is the basis of the planar wavefront models developed in this chapter.

B. PLANE EQUATIONS

A plane in space is fully defined by identifying any point (X0,Y0,Z0) on the plane, and also a vector (C1,C2,C3) of unit length which is perpendicular to that plane. The vector is called the unit normal vector for that plane. Then any point (X,Y,Z) on that plane must satisfy the equation of the plane, namely (3.1).

$$C_1 (X - X_0) + C_2 (Y - Y_0) + C_3 (Z - Z_0) = 0 \quad (3.1)$$

The perpendicular distance from the plane to any point (X1,Y1,Z1) not on the plane is the absolute value of (3.2).

$$C_1 (X_1 - X_0) + C_2 (Y_1 - Y_0) + C_3 (Z_1 - Z_0) \quad (3.2)$$

C. BASIC MODEL EQUATIONS

Consider a coordinate system whose origin is at hydrophone c, and whose axes are aligned with the three array arms. Let C1, C2 and C3 be the direction cosines on the X, Y and Z axes respectively for the vector from the origin to the apparent sound source position. Then (C1,C2,C3) is itself a vector, of unit length, which is perpendicular to the planar wavefront emanating from the sound source. Therefore (C1,C2,C3) can be used as the normal vector for the wavefront plane.

In the coordinate system referenced to the c-phone as the origin, the acoustic center has coordinates (D,D,D)/2, where D is the length of an array arm. When the soundwave plane arrives at the acoustic center, it will have the equation (3.3).

$$C_1 X + C_2 Y + C_3 Z = D (C_1 + C_2 + C_3) / 2 \tag{3.3}$$

The x-phone has coordinates (D,0,0), and the distance between it and the soundwave plane at the acoustic center is (3.4), which then simplifies to (3.5).

$$C_1 (D/2 - D) + C_2 (D/2 - 0) + C_3 (D/2 - 0) \tag{3.4}$$

$$D (-C_1 + C_2 + C_3) / 2 \tag{3.5}$$

This distance is measured in a direction perpendicular to the wavefront plane, and so is measured in the direction of travel of the soundwave. Therefore the same distance is also equal to (3.6).

$$V (T_x - T) \tag{3.6}$$

In (3.6) V is the velocity of sound at the array, Tx is the time of arrival of the sound wave at the x-phone, and T is the time of arrival of the sound wave at the acoustic center. The term "distance" is used loosely in (3.5) and (3.6), because these quantities may be negative. The true distances are the absolute values of these quantities. Since the next step is to equate these two distances, it is only necessary to show that these two quantities always have the same sign. There are two cases to consider, depending on whether the first component of the apparent position is positive (X>0), or negative (X<0). Let (X',0,0) be the intersection of the X axis with the wavefront plane as it passes through the acoustic center. Then (3.3) can be used to solve for X', namely

$$X' = D\ (C1 + C2 + C3)\ /\ 2\ C1.$$

Now consider the case where X>0. Then C1>0 also, and if (3.6) is positive, then

$$Tx > T$$

which implies that the wave arrives at the acoustic center before it arrives at the the x hydrophone. Therefore, since X>0, the plane at the acoustic center will intersect the X axis at a point beyond the x hydrophone, or X'>D, and hence

$$\begin{aligned} X' > D \quad &\Rightarrow \quad (C1 + C2 + C3)/\ 2\ C1 \quad > \quad 1 \\ &\Rightarrow \quad C1 + C2 + C3 \quad > 2\ C1 \qquad \text{(since } C1>0\text{)} \\ &\Rightarrow \quad -C1 + C2 + C3 \quad > \quad 0 \\ &\Rightarrow \quad \text{(3.5) is positive .} \end{aligned}$$

A parallel argument applies for the case of X>0, thus establishing that (3.5) and (3.6) always have the same sign. Equation (3.7) is the result of equating these two quantities.

$$-C_1 + C_2 + C_3 + (\ 2\ V\ T\ /\ D\ ) - (\ 2\ V\ T_1\ /\ D\ ) \ = \ 0 \qquad (3.7)$$

For convenience, let $K = 2V/D$, and then apply the same logic to the distances of the y, z, and c hydrophones from the wavefront plane as it passes through the acoustic center, to obtain the equations of (3.8).

$$
\begin{aligned}
C_1 - C_2 - C_3 - K\,T + K\,T_x &= 0 \\
-C_1 + C_2 - C_3 - K\,T + K\,T_y &= 0 \\
-C_1 - C_2 + C_3 - K\,T + K\,T_z &= 0 \\
C_1 + C_2 + C_3 + K\,T - K\,T_c &= 0
\end{aligned}
\qquad (3.8)
$$

The system (3.8) is four equations in four unknowns, and is the planar model's version of the equations (1.1). The unknowns in (3.8) are C1, C2, C3 and T. However there is the additional constraint that C1, C2, and C3 are direction cosines, and therefore the direction cosines constraint (3.9) is a fifth equation, creating a a system of five equations in four unknowns.

$$C_1^2 + C_2^2 + C_3^2 = 1 \qquad (3.9)$$

Generally there is no set of values (C1,C2,C3,T) which will satisfy all five equations at once. This is because of the realities of a nonplanar wavefront and the presence of timing errors. The next section developes a method that produces set of values for the unknowns which is intended to satisfy those equations reasonably well.

D. MINIMIZATION OF SUM OF SQUARED ERRORS

Let Ei, (i=1,2,3,4) be the value of the left hand side of the i-th equation of (3.8). Then Ei measures the amount of error in the i-th equation caused by the chosen solution. It is impossible to have Ei=0 for all i=1,2,3,4. However some compromise may be made. Specifically the compromise chosen here is the classic minimization of (3.10), the sum of squared errors.

$$E_1^2 + E_2^2 + E_3^2 \tag{3.10}$$

Minimization is to be done subject to the direction cosine constraint (3.9). Application of the Lagrange multiplier technique [Ref. 5 p.55] calls for the minimization of (3.11) over all possible choices of C1, C2, C3, T and lambda.

$$L = \sum_{i=1}^{4} E_i^2 - \lambda \left(\sum_{i=1}^{3} C_i^2 - 1 \right) \tag{3.11}$$

Taking the partial derivative of L with respect to T yields (3.12).

$$\begin{aligned} \frac{\partial L}{\partial T} &= \sum_{i=1}^{4} (-2K\,E_i) = -2K\,(E_1 + E_2 + E_3 + E_4) \\ &= 2\,K \left[-2K\,T + K\,(T_1 + T_2 + T_3 - T_4) \right] \end{aligned} \tag{3.12}$$

Equating (3.12) to zero and solving for T yields immediately the appropriate estimate (3.13) of T, the ray transit time.

$$T = \frac{1}{2}\,(T_1 + T_2 + T_3 - T_4) \tag{3.13}$$

The partial derivative of L with respect to C1 is (3.14).

$$\begin{aligned} \frac{\partial L}{\partial C_1} &= 2\,(E_1 - E_2 - E_3 + E_4) - 2\lambda C_1 \\ &= 2 \left[4C_1 + 2KT + K\,(T_1 - T_2 - T_3 - T_4) - \lambda C_1 \right] \end{aligned} \tag{3.14}$$

If (3.13) is used for T in (3.14), then (3.15) results.

$$\frac{\partial L}{\partial C_1} = 2 \left[(4 - \lambda)\,C_1 + 2\,K\,(T_1 - T_4) \right] \tag{3.15}$$

If the same procedure is used for the partial derivatives of L with respect to each of C1, C2 and C3, and all are equated to zero, then the results are (3.16).

$$(4 - \lambda) C_i = 2 K (T_4 - T_i) \qquad i = 1,2,3 \qquad (3.16)$$

In order to solve for the Lagrange multiplier lambda, square both sides of the three equations of (3.16), and add the resulting equations together. Then use the sum of squares constraint (3.9) and solve for lambda to yield (3.17).

$$\lambda = 4 \pm 2 K \left[\sum_{i=1}^{3} (T_4 - T_i)^2 \right] \qquad (3.17)$$

Substitute (3.17) in (3.16) and simplify to obtain the appropriate estimate of Ci, namely (3.18).

$$C_i = \frac{T_4 - T_i}{\sqrt{\sum_{j=1}^{3} (T_4 - T_j)^2}} \qquad i = 1,2,3 \qquad (3.18)$$

The choice of sign in (3.18) is determined by the fact that Ci is positive if and only if the sound wave arrives at the i-th phone before it arrives at the c-phone, which in turn implies that (T4-Ti)>0.

E. THE LEAST SQUARES METHOD

In summary, the first alternative method for determining an apparent position starts with estimation of the ray transit time to the acoustic center by (3.19). Then the apparent position estimates are computed using (3.20).

$$T = (T_1 + T_2 + T_3 - T_4) / 2 \qquad (3.19)$$

$$X = \frac{V\,T\,(T_4 - T_1)}{\sqrt{\sum_{j=1}^{3}(T_4 - T_j)^2}} \qquad Y = \frac{V\,T\,(T_4 - T_2)}{\sqrt{\sum_{j=1}^{3}(T_4 - T_j)^2}} \tag{3.20}$$

$$Z = \frac{V\,T\,(T_4 - T_3)}{\sqrt{\sum_{j=1}^{3}(T_4 - T_j)^2}}$$

This method shall be hereafter referred to as the 'Least Squares Method', or 'L.S.' for short. The apparent advantages of the L.S. method are that:

1. all four hydrophone times have equal weight in a simple expression for the ray transit time T, rather than using an expression so heavily dependent on Tc as in the NAVY and NAVY_A methods;
2. the differences of squared time values which appear in the solutions of the NAVY methods are avoided in the L.S. method, thereby lessening the tendency toward computational roundoff problems;
3. the direction cosines already add to unity when squared, requiring no arbitrary adjustment; and
4. computation of the initial angle allows cancellation of several terms, resulting in the simple expression (3.21).

$$A = \arcsin\left((T_4 - T_3) \Big/ \sqrt{\sum_{j=1}^{3}(T_4 - T_j)^2} \right) \tag{3.21}$$

F. BIAS IN THE LEAST SQUARES METHOD

Unfortunately a potentially serious problem exists with the L.S. method. That concerns the consequences of the assumption of a planar wavefront. The effect is difficult

to derive explicitly because it is difficult to determine the true shape of a wavefront corresponding to a nonconstant velocity profile. However if it is assumed that a spherical assumption is more accurate the planar assumption, then the bias can be estimated roughly, and then subtracted from the original L.S. position estimate. This is still a difficult problem because, as previously discussed, the four basic spherical equations themselves have no exact solution.

Nevertheless, as a rough estimate of the bias, the following procedure is used. First estimate the apparent position by the L.S. method. Then calculate the straight line distances from that position to each of the four array hydrophones. Divide those distances by the velocity of sound at the array to obtain the corresponding times. Use these times to recalculate the apparent position using the L.S. method again. The difference between the original and recalcualted L.S. positions roughly measures the error that would be made by the L.S. method when it is applied to the time values which correspond to a spherically spreading sound wave whose source is in the vicinity of the original apparent position. Therefore this difference can be used as a bias vector which can be subtracted from the original L.S. solution. This bias correction is the basis of the method set forth in the next section.

G. THE LEAST SQUARES CORRECTED METHOD

The second alternative method for estimating an apparent position is as follows:

1. calculate the apparent position P1 by using the L.S. method;
2. calculate the distances from that position to the four hydrophones, and convert them to times by dividing by the speed of sound at the array;

3. use the new times to calculate a new apparent position P2, using the L.S. method;
4. calculate the difference vector P2-P1 (see figure 3.1), and subtract it from the original position P1 to obtain the corrected position P

$$P = P1 - (P2 - P1) = 2 P1 - P2$$

5. finally adjust the ray transit time T calculated for the original position P1, by using the proportional transformation:

$$T' = T * R / R1$$

where R is the range to the new position P, and R1 is the range to the original position P1.

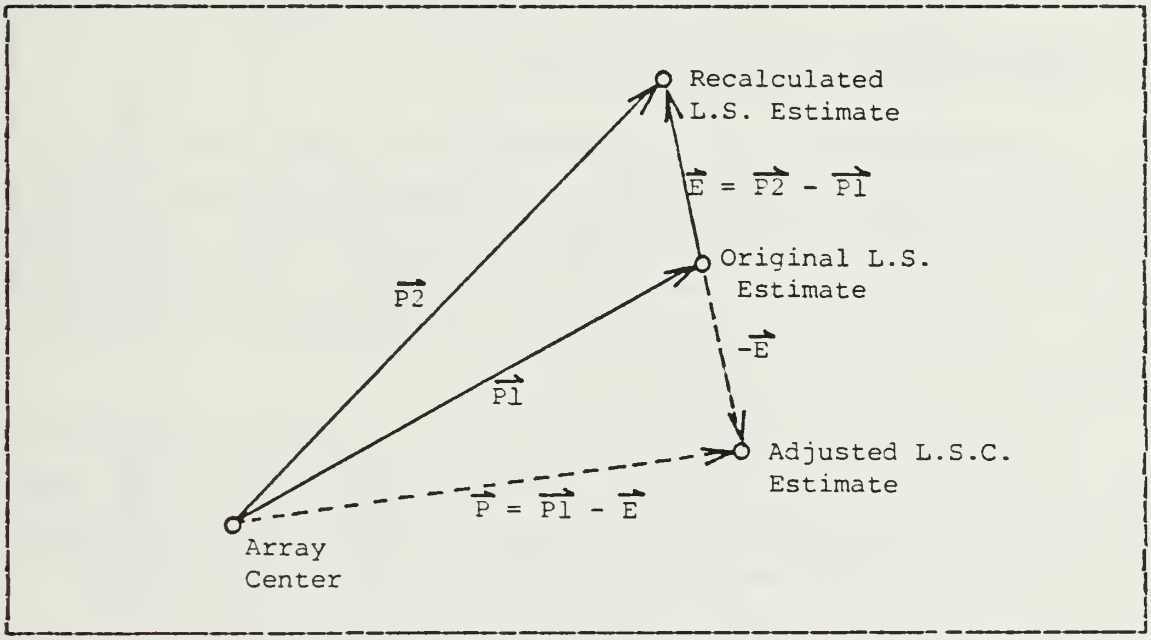


Figure 3.1 Bias Adjustment for the L.S. Method.

This method shall hereafter be referred to as the 'Least Squares Corrected Method', or L.S.C. for short. The properties of the L.S.C. method, like those of the NA/Y_A method, are not well understood at this time. It is offered only as

an alternative which may combine the beneficial properties of the L.S. and NAVY methods, namely that it will:

1. estimate a ray transit time value equally dependent on all four data times, thereby smoothing out excessive error in any one of the time values; and
2. reflect the spherical wave assumption, believed to provide a more accurate description than the planar assumption.

For targets at a range of 3000 feet, the length of the error vector P2-P1 varies from 0 to as much as 10 or 11 feet (see Table II). The error vector lengths seem to be dependent on both azimuth and elevation angles of the target from the array. These patterns indicate a potential for further investigation to relate estimation errors to such variables.

H. MAXIMUM LIKELIHOOD CALCULATIONS

One shortcoming of all methods described thus far is that none of them can be used to produce an estimate of the underlying error in the time data values. The method set forth in this section is a first attempt to estimate that noise, and is again based on the assumption of a planar wavefront.

Let T_i be the time recorded from by i-th hydrophone. Let U_i be the true time which, under absolutely error free conditions, would have been recorded at the i-th hydrophone. An assumption of Gaussian noise is now made, namely that

$$T_i = U_i + E_i$$

where the E_i are independent identically distributed normal random variables with mean zero and variance S^2. Therefore the T_i $(i=1,2,3,4)$ are also normally distributed with the same variance, but with means U_i $(i=1,2,3,4)$.

TABLE II

Error Vector Lengths for the L.S.C. Method

Tabled entries are lengths in feet of bias vectors.

Angles are in multiples of π .

ELEVATION ANGLE ↓	← AZIMUTH ANGLE → .0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
.50	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
.48	.8	.8	.8	.8	.8	.8	.9	.9	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	.9	.9	.8
.46	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.8	2.0	2.1	2.2	2.2	2.1	2.1	2.2	2.2	2.1	2.0	1.8	1.6	1.6
.44	2.2	2.2	2.3	2.3	2.2	2.2	2.3	2.6	3.0	3.2	3.3	3.3	3.2	3.2	3.3	3.3	3.2	3.0	2.6	2.3	2.2
.42	2.6	2.7	2.8	2.8	2.7	2.6	2.8	3.3	3.9	4.3	4.5	4.4	4.3	4.3	4.4	4.5	4.3	3.9	3.3	2.8	2.6
.40	2.8	3.0	3.2	3.2	3.0	2.8	3.1	3.9	4.8	5.4	5.6	5.5	5.3	5.3	5.5	5.6	5.4	4.8	3.9	3.1	2.8
.38	2.9	3.1	3.4	3.4	3.1	2.9	3.3	4.4	5.6	6.5	6.7	6.6	6.3	6.3	6.6	6.7	6.5	5.6	4.4	3.3	2.9
.36	2.8	3.1	3.4	3.4	3.1	2.8	3.4	4.8	6.4	7.4	7.8	7.5	7.1	7.1	7.5	7.8	7.4	6.4	4.8	3.4	2.8
.34	2.5	2.9	3.3	3.3	2.9	2.5	3.3	5.2	7.0	8.3	8.7	8.4	7.9	7.9	8.4	8.7	8.3	7.0	5.2	3.3	2.5
.32	2.1	2.6	3.1	3.1	2.6	2.1	3.2	5.4	7.5	8.9	9.4	9.1	8.5	8.5	9.1	9.4	8.9	7.5	5.4	3.2	2.1
.30	1.6	2.2	2.8	2.8	2.2	1.6	3.0	5.7	8.0	9.5	10.0	9.6	8.9	8.9	9.6	10.0	9.5	8.0	5.7	3.0	1.6
.28	1.0	1.9	2.3	2.3	1.9	1.0	2.9	5.9	8.3	9.9	10.4	10.0	9.2	9.2	10.0	10.4	9.9	8.3	5.9	2.9	1.0
.26	.4	1.6	1.9	1.9	1.6	.4	2.9	6.1	8.6	10.1	10.6	10.1	9.3	9.3	10.1	10.6	10.1	8.6	6.1	2.9	.4
.24	.4	1.5	1.4	1.4	1.5	.4	3.1	6.3	8.7	10.1	10.6	10.1	9.2	9.2	10.1	10.6	10.1	8.7	6.3	3.1	.4
.22	1.0	1.7	1.1	1.1	1.7	1.0	3.4	6.6	8.8	10.0	10.3	9.9	8.9	8.9	9.9	10.3	10.0	8.8	6.6	3.4	1.0
.20	1.6	2.1	1.0	1.0	2.1	1.6	3.8	7.0	8.9	9.7	9.9	9.5	8.5	8.5	9.5	9.9	9.7	8.9	7.0	3.8	1.6
.18	2.1	2.5	1.3	1.3	2.5	2.1	4.2	7.4	9.0	9.2	9.3	8.9	7.9	7.9	8.9	9.3	9.2	9.0	7.4	4.2	2.1
.16	2.5	2.9	1.6	1.6	2.9	2.5	4.6	7.8	9.0	8.7	8.5	8.2	7.2	7.2	8.2	8.5	8.7	9.0	7.8	4.6	2.5
.14	2.8	3.2	1.9	1.9	3.2	2.8	5.0	8.2	9.1	8.1	7.6	7.4	6.4	6.4	7.4	7.6	8.1	9.1	8.2	5.0	2.8
.12	2.9	3.4	2.1	2.1	3.4	2.9	5.3	8.7	9.2	7.5	6.6	6.5	5.5	5.5	6.5	6.6	7.5	9.2	8.7	5.3	2.9
.10	2.8	3.5	2.2	2.2	3.5	2.8	5.5	9.0	9.4	6.9	5.5	5.6	4.6	4.6	5.6	5.5	6.9	9.4	9.0	5.5	2.8
.08	2.6	3.4	2.2	2.2	3.4	2.6	5.7	9.4	9.5	6.3	4.3	4.7	3.7	3.7	4.7	4.3	6.3	9.5	9.4	5.7	2.6
.06	2.2	3.3	2.1	2.1	3.3	2.2	5.7	9.7	9.7	5.9	3.1	4.0	2.9	2.9	4.0	3.1	5.9	9.7	9.7	5.7	2.2
.04	1.6	3.1	1.9	1.9	3.1	1.6	5.7	9.9	9.8	5.6	2.0	3.3	2.2	2.2	3.3	2.0	5.6	9.8	9.9	5.7	1.6
.02	.8	2.9	1.7	1.7	2.9	.8	5.7	10.0	9.9	5.5	.9	3.0	1.7	1.7	3.0	.9	5.5	9.9	10.0	5.7	.8
.00	.1	2.8	1.6	1.6	2.8	.1	5.6	10.0	9.9	5.5	.1	2.9	1.6	1.6	2.9	.1	5.5	9.9	10.0	5.6	.1

Now let Cj be the j-th direction cosine (j=1, 2,3). Then geometrically, using the assumption of a planar wavefront, Cj is given by (3.22), where V is the speed of sound at the array, and D is the array dimension (30 feet).

$$C_j = V (U_4 - U_j) / D \qquad (3.22)$$

Letting U = U4, then (3.22) can be solved for each Uj in terms of U and Cj, as in (3.23).

$$U_j = U - D C_j / V \qquad (3.23)$$

The probability density of each time value Ti is given by (3.24).

$$f_i(T_i) = \frac{1}{\sqrt{2\pi}\ S} \exp\left[\frac{-(T_i - U_i)^2}{2 S^2}\right] \qquad (3.24)$$

Using (3.23) in (3.24), and multiplying the four densities together, the likelihood function (3.25) is formed.

$$L_0 = \left(\frac{1}{\sqrt{2\pi}\ S}\right)^4 \exp\left[\frac{-1}{2 S^2} \sum_{i=1}^{4} (T_i - U + DC_i/V)^2\right] \qquad (3.25)$$

In (3.25) C4 has been used for notational convenience, and is defined to be zero. Then the log-likelihood function is formed by taking natural logs of (3.25), yielding (3.26).

$$L_1 = -2 \ln(2\pi) - 4 \ln(S) - \frac{1}{2 S^2} \sum_{i=1}^{4} (T_i-U+DC_i/V)^2 \qquad (3.26)$$

Since the values of the Ci are to be direction cosines, the usual direction cosines constraint must be added to L1 to form the Lagrangian function L2 given in (3.27).

$$L_2 = L_1 - \lambda \left[\sum_{i=1}^{3} (C_i^2) - 1\right] \qquad (3.27)$$

The objective is to maximize L2 over the possible choices of C1, C2, C3, U, S and lambda. Ignoring S for the moment, maximization of L2 can be acheived by minimization of L given in (3.28).

$$L = \frac{1}{2\,S^2} \sum_{i=1}^{4} (T_i - U + DC_i/V)^2 + \lambda\left[\sum_{i=1}^{3} (C_i^2) - 1\right] \qquad (3.28)$$

Take partial derivatives of L to get (3.29) and (3.30).

$$\frac{\partial L}{\partial C_i} = \frac{2D}{V} (T_i - U + DC_i/V) - 2\lambda C_i \qquad (3.29)$$

$$\frac{\partial L}{\partial U} = 2\left[\sum_{i=1}^{4} T_i - 4U + \frac{D}{V} \sum_{i=1}^{3} C_i\right] \qquad (3.30)$$

Equate (3.30) to zero and solve for U to obtain (3.31), the maximum likelihood estimate for the time at the c hydrophone.

$$U = \left(\sum_{j=1}^{4} T_j + \frac{D}{V} \sum_{j=1}^{3} C_j\right) \Big/ 4 \qquad (3.31)$$

Equate the three equations of (3.29) to zero, and multiply each equation by Ci respectively, to obtain (3.32).

$$\lambda C_i^2 = \frac{D}{V}\left(T_i\, C_i - UC_i + \frac{D}{V} C_i^2\right) \qquad i = 1,2,3 \qquad (3.32)$$

Add the three equations of (3.32) together, and use the direction cosine constraint to obtain (3.33).

$$\lambda = \frac{D}{V}\left[\sum_{i=1}^{3} T_i C_i - U \sum_{i=1}^{3} C_i + \frac{D}{V}\right] \qquad (3.33)$$

Equation (3.34) results when (3.29) is equated to zero.

$$C_i = \frac{T_i - U}{\frac{V}{D}\lambda - \frac{D}{V}} \qquad i = 1,2,3 \tag{3.34}$$

Substitute (3.33) into (3.34) to obtain (3.35), the maximum likelihood estimate of the direction cosines, where U is the estimate calculated by (3.31).

$$C_i = \frac{T_i - U}{\sum_{j=1}^{3} T_j C_j - U \sum_{j=1}^{3} C_j} \tag{3.35}$$

As the reader will perhaps have noticed, the equations of (3.35) define each of the unknowns Ci in terms of all three unknowns. Such a structure suggests that (3.35) can be used as an iteration function. That is, if reasonable initial values are used for the three unknowns in the right hand side of (3.35) then new values are produced. Repeat the process using the new values until the answer stabilizes within acceptable tolerances. Although convergence to the correct solution is not guaranteed, the method has never failed for the equations of (3.35). Unlike the L.S. method, (3.35) does not have any known closed form solution.

Returning to the standard deviation S, take the partial derivative of (3.27) with respect to S to get (3.36).

$$\frac{\partial L_2}{\partial S} = \frac{-4}{S} + \frac{1}{S^3} \sum_{i=1}^{4} (T_i - U + \frac{D}{V} C_i)^2 \tag{3.36}$$

Multiply (3.36) by S^3 and solve for S^2 to get the maximum likelihood estimate of the variance, given by (3.37).

$$S^2 = \frac{1}{4}\left[\sum_{i=1}^{4} (T_i - U) + \frac{D}{V} \sum_{i=1}^{3} C_i\right] \tag{3.37}$$

I. THE MAXIMUM LIKELIHOOD PLANAR METHOD

In summary, the third method for estimation of an apparent position is as follows:

1. let U = T4 initially;
2. use the L.S. method to calculate the initial values for Ci, (i=1,2,3), using (Ee321);
3. use U and Ci (i=1,2,3) in the right hand side of (3.35) to obtain new estimates for Ci (i=1,2,3);
4. recalculate U, using (3.31);
5. reiterate steps 3 and 4 until the values Ci (i=1,2,3) and U converge within acceptable tolerances;
6. calculate S^2, using (3.37);
7. calculate the estimated apparent position relative to the c-phone, using (3.38);

$$X_c = V\,U\,C_1 \qquad Y_c = V\,U\,C_2 \qquad Z_c = V\,U\,C_3 \tag{3.38}$$

8. lastly translate this solution and its corresponding time estimate to a solution and time relative to the acoustic center, using (3.39), where R and Rc are as defined by (1.4) in Chapter I.

$$X = X_c + D/2 \qquad Y = Y_c + D/2 \qquad Z = Z_c + D/2$$
$$T = U\,R\,/\,R_c \tag{3.39}$$

This method shall be hereafter referred to as the 'Maximum Likelihood Planar Method', or M.L.P. for short. Originally the hopes for this method were rather high, especially since it was the first method to produce a variance estimate. However subsequent experience with the method indicates that it probably suffers significantly from at least two factors:

1. the planar wavefront assumption probably builds in a position bias as in the case of the L.S. method; and

2. the variance estimate is inflated since part of the noise being measured is due to the inadequacy of the planar assumption.

J. COMPUTATIONAL EXAMPLE

For a quick comparison, the three methods developed in this chapter are now applied to the example which was used in Chapter II. The EXACT results calculated previously are included in Table III for comparison.

TABLE III

Single Example Comparison of Planar Wavefront Methods

Method	Transit Time T (secs)	Elev. Angle A (degs)
L.S.	0.67529319	15.22798
L.S.C.	0.67527149	15.26372
M.L.P.	0.67529007	15.10437
EXACT	0.67527043	15.27002

	Apparent Position Estimate
L.S.	(1003.809 , 3019.751 , 866.126)
L.S.C.	(1006.089 , 3018.266 , 868.235)
M.L.P.	(997.722 , 3023.685 , 859.355)
EXACT	(1005.966 , 3017.899 , 868.555)

As can be seen easily, the M.L.P. method fares rather poorly in all regards, even worse than the L.S. method. This will be confirmed by the evaluations made in Chapter V. Also worthy of note is the apparent tendency of the L.S.C.

method to correct the L.S. method back toward the exact solution. The evaluations of Chapter V will confirm that the L.S.C. method almost always yields a better solution than the original L.S. solution.

IV. A SPHERICAL WAVEFRONT MODEL

A. THE SPHERICAL WAVEFRONT ASSUMPTION

All the models developed in Chapter III are limited primarily by the assumption that the wavefront is planar upon arrival at the hydrophone array. In this chapter a model is developed under the assumption that the wavefront is spherical upon arrival at the array. If the sound velocity profile were constant with depth then the spherical model would be exact. This is of course not the case, but it is suspected that the wavefront is better modelled as a sphere than as a plane because that small piece of the wavefront which passes through the 30 foot cube spanned by the array is locally spherical. That is because every part of that piece travelled through approximately the same regions of water, experiencing the same general raybending patterns.

The spherical assumption is accurate if and only if the speed of sound is constant over the ray path, and consequently the original four spherical equations apply once again. They were:

$$\begin{aligned}
(X - D/2)^2 + (Y + D/2)^2 + (Z + D/2)^2 &= V^2T_x^2 \\
(X + D/2)^2 + (Y - D/2)^2 + (Z + D/2)^2 &= V^2T_y^2 \\
(X + D/2)^2 + (Y + D/2)^2 + (Z - D/2)^2 &= V^2T_z^2 \\
(X + D/2)^2 + (Y + D/2)^2 + (Z + D/2)^2 &= V^2T_c^2
\end{aligned} \qquad (4.1)$$

It has been stressed previously that there is no exact solution (X,Y,Z) satisfying all four equations (4.1). That is because the time values on the right hand side correspond to the reality of a variable velocity profile. However, if the spherical wavefront assumption is to be accurate, then a

constant velocity is the assumed case, and any innacuracies in the time values are regarded as due to timing errors only. Therefore, under the spherical assumption, if the true time values Ui (i=1,2,3,4) were known and substituted into (4.1), an exact solution to the overdetermined system would be realized. In that case a solution to any three of the equations would also be equal to that unique exact solution. In particular, the NAVY solution of Chapter II would be the true solution. Therefore in terms of the coordinate system referenced to the c hydrophone, X would be given by (4.2).

$$X = \frac{D}{2} + \frac{V^2}{2D} (U_4^2 - U_1^2) \tag{4.2}$$

However, X is also given by (4.3), where C1 is the direction cosine along the X axis of the vector from the c hydrophone to the sound source.

$$X = V U_4 C_1 \tag{4.3}$$

The time value of interest at the moment is U4, the time at the c hydrophone. Therefore let U = U4 for clarity of presentation, and then equate the expressions in (4.2) and (4.3) in order to solve for U. If this same logic is also applied to the similar expressions for the distances to the y and z hydrophones, the results are (4.4).

$$U_i = \sqrt{U^2 - (2DUC_i/V) + (D/V)^2} \qquad i = 1,2,3 \tag{4.4}$$

The expression (4.4) will be useful in the development of the model of this chapter.

B. LEAST SQUARES MODELS

A direct approach might be to apply the least squared error technique to the spherical equations, in a manner

paralleling that used on the four planar equations of the L.S. model in Chapter III. However the formulae and equations that result are exceedingly complex, involve fourth degree powers of the data values Ti, and have thus far defied all solution attempts. Therefore this idea was abandoned in favor of the maximum likelihood approach which follows.

C. MAXIMUM LIKELIHOOD COMPUTATIONS

As in the M.L.P. model, Gaussian noise is assumed for the time data values. Therefore

$$Ti = Ui + Ei \qquad i = 1,2,3$$

where the Ei are independent identically distributed normal random variables with mean zero and variance S^2. The density of each Ti is therefore (4.5).

$$f_i(T_i) = \frac{1}{\sqrt{2\pi}\, S} \exp\left[\frac{-1}{2\, S^2} (T_i - U_i)^2\right] \quad i = 1,2,3,4 \qquad (4.5)$$

To form the likelihood function, multiply the four densities together, to obtain (4.6).

$$L_0 = \left(\frac{1}{\sqrt{2\pi}\, S}\right)^4 \exp\left[\frac{-1}{2\, S^2} \sum_{i=1}^{3} (T_i - U_i)^2 + (T_4 - U)^2\right] \qquad (4.6)$$

Form the log likelihood function by taking natural logarithms of (4.6) to obtain L1 of (4.7).

$$L_1 = -2 \ln(2\pi) - 4 \ln(S) - \frac{1}{2\, S^2}\left[\sum_{i=1}^{3} (T_i - U_i)^2 + (T_4 - U)^2\right] \qquad (4.7)$$

In order to maximize L1 with respect to C1, C2, C3 and U, it is sufficient to minimize L2 of (4.8), where a substitution for Ui has been made using (4.4).

$$L_2 = (T_4-U)^2 + \sum_{i=1}^{3}\left[T_i - \sqrt{U^2 - (2DUC_i/V) + (D/V)^2}\right]^2 \qquad (4.8)$$

Now add the usual direction cosines constraint and form the Lagrangian function L of (4.9).

$$L = L_2 - \lambda\left[\sum_{i=1}^{3} C_i^2 - 1\right] \qquad (4.9)$$

For notational convenience, let Ki be given by (4.10).

$$K_i = U^2 - (2\ D\ U\ C_i\ /\ V) + (D/V)^2 \qquad i = 1,2,3 \qquad (4.10)$$

Take the partial derivative of L with respect to Ci to get (4.11).

$$\frac{\partial L}{\partial C_i} = \left(\frac{2\ (T_i - K_i)}{-2\ K_i}\right)\left(\frac{-2\ D\ U}{V}\right) - 2\lambda C_i \qquad i = 1,2,3 \qquad (4.11)$$

Simplify (4.11) and equate to zero to yield (4.12).

$$C_i = \frac{D\ U}{\lambda V}\left(\frac{T_i - \sqrt{K_i}}{\sqrt{K_i}}\right) \qquad i = 1,2,3 \qquad (4.12)$$

Multiply the three equation in (4.12) by Ci (i=1,2,3) respectively, and add them together. Then use the constraint on the sum of squares of the direction cosines to obtain (4.13).

$$\lambda = \frac{D\ U}{V}\sum_{i=1}^{3} C_i\left(\frac{T_i - \sqrt{K_i}}{\sqrt{K_i}}\right) \qquad (4.13)$$

Substitute (4.13) into (4.12) to get the maximum likelihood estimate for the direction cosines, as in (4.14).

$$C_i = \frac{T_i - \sqrt{K_i}}{\sqrt{K_i}\sum_{j=1}^{3} C_j\left(\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}}\right)} \qquad i = 1,2,3 \tag{4.14}$$

Now take the partial derivative of L with respect to U, as in (4.15).

$$\frac{\partial L}{\partial U} = 2\sum_{i=1}^{3}\left[\left(\frac{T_i - \sqrt{K_i}}{-2\sqrt{K_i}}\right)(2U - 2DC_i/V)\right] + 2\,(U - T_4) \tag{4.15}$$

Equate (4.15) to zero and solve for U to obtain the maximum likelihood estimate of the time value (4.16) at the c hydrophone.

$$U = \frac{T_4 - \frac{D}{V}\sum_{j=1}^{3} C_j\left(\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}}\right)}{1 - \sum_{j=1}^{3}\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}}} \tag{4.16}$$

Finally take the partial derivative of L1 in (4.7) with respect to S. Equate it to zero and solve to get (4.17), the maximum likelihood estimate of the variance.

$$S^2 = \frac{1}{4}\left[\sum_{j=1}^{3}(T_j - K_j)^2 + (T_4 - U)^2\right] \tag{4.17}$$

D. SOLUTION BY A MODIFICATION OF NEWTON'S METHOD

The solutions given by equations (4.14), (4.16), and (4.17) once again form a set of equations which would seem to be solvable by natural iteration as in the M.L.P. method. Unfortunately this time the technique fails to converge. A mathematical tool is needed which is stronger than natural iteration. What is used is a modified four dimensional

version of Newton's Method [Ref. 5 p.47] to search for the roots of a set of four equations.

The objective is to determine the values C1, C2, C3 and U which satisfy (4.14) and (4.16). For further notational convenience define the values M and N as in (4.18) and (4.19).

$$M = \sum_{j=1}^{3} C_j \left(\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}} \right) \tag{4.18}$$

$$N = \sum_{j=1}^{3} \left(\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}} \right) \tag{4.19}$$

Given those definitions of M and N, then the equations whose roots are desired can be simplified to (4.20).

$$\begin{aligned} C_i &= h_i(C_1,C_2,C_3,U) = \frac{T_i - \sqrt{K_i}}{\sqrt{K_i}\ M} \qquad i = 1,2,3 \\ U &= h_4(C_1,C_2,C_3,U) = \frac{T_4 - \frac{D\ M}{V}}{1 - N} \end{aligned} \tag{4.20}$$

Now define error functions as in (4.21). These evaluate the amount of error in each of the equations (4.20) for any set of values for (C1,C2,C3,U).

$$\begin{aligned} g_i &= C_i - h_i \qquad i = 1,2,3 \\ g_4 &= U - h_4 \end{aligned} \tag{4.21}$$

Let G be the four dimensional column vector (g1,g2,g3,g4)'. Finally let GP be the matrix of partial derivatives of G, as in (4.22).

Newton's Method in four dimensions says that if $\underline{X}(n)$ is a four dimensional vector holding the current approximate roots C1,C2,C3 and U, then $\underline{X}(n+1)$ will be an improved answer, where $\underline{X}(n+1)$ is given by (4.23).

$$GP\,(C_1,C_2,C_3,U) = \begin{bmatrix} \frac{\partial g_1}{\partial c_1} & \frac{\partial g_1}{\partial c_2} & \frac{\partial g_1}{\partial c_3} & \frac{\partial g_1}{\partial c_4} \\ \frac{\partial g_2}{\partial c_1} & \frac{\partial g_2}{\partial c_2} & \frac{\partial g_2}{\partial c_3} & \frac{\partial g_2}{\partial c_4} \\ \frac{\partial g_3}{\partial c_1} & \frac{\partial g_3}{\partial c_2} & \frac{\partial g_3}{\partial c_3} & \frac{\partial g_3}{\partial c_4} \\ \frac{\partial g_4}{\partial c_1} & \frac{\partial g_4}{\partial c_2} & \frac{\partial g_4}{\partial c_3} & \frac{\partial g_4}{\partial c_4} \end{bmatrix} \tag{4.22}$$

$$\underline{X}_{n+1} = \underline{X}_n - \left[GP(\underline{X}_n)\right]^{-1} \cdot G(\underline{X}_n) \tag{4.23}$$

Of course it is necessary to calculate the derivatives held in the matrix GP in order to use (4.23). Those derivatives are given in Appendix B.

Unfortunately when multidimensional versions of Newton's Method are applied, there is often a tendency for the method to converge slowly, or even diverge. This is because it tends to overshoot the best answer for each iteration. To alleviate this problem, a modification is made to the method. At the end of each Newton iteration, prior to proceeding with the next iteration, a Golden Section Search [Ref. 6] is performed to find the best possible answer in the direction of the new iterative solution. Specifically the line in four dimensional space from $\underline{X}(n-1)$ of the previous iteration to $\underline{X}(n)$ of the present iteration is searched for the best answer. The definition of the 'best' answer is that point along the search line which minimizes the sum of squared error functions, namely (4.24).

$$\sum_{i=1}^{4} g_i^2 \tag{4.24}$$

The current iterative solution $\underline{X}(n)$ is then given the value of the minimizing point resulting from the Golden Section Search. Then the next iteration of Newton's Method is performed, along with another Golden Section Search to find the next iterative solution $\underline{X}(n+1)$.

E. THE MAXIMUM LIKELIHOOD METHOD

In summary, the fourth and final alternative method to estimate apparent positions is as follows:

1. let U = T4 initially;
2. use the L.S. method (&e321) to initially estimate the values of Cj (j=1,2,3);
3. set $\underline{X}(1)$ = (C1,C2,C3,U);
4. initialize the Newton Iteration counter : I = 1 ;
5. calculate the values Ki, M and N in accordance with equations (4.10), (4.18) and (4.19);
6. calculate the error function vector $G(\underline{X}(I))$, using (4.20) and (4.21);
7. calculate the derivative matrix $GP(\underline{X}(I))$, using the results in Appendix B;
8. invert $GP(\underline{X}(I))$;
9. calculate the new Newton estimate $\underline{X}(I+1)$ from (4.23);
10. perform a Golden Section Search along the line between $\underline{X}(I)$ and $\underline{X}(I+1)$ to find the point which minimizes (4.24);
11. let $\underline{X}(I+1)$ be equal to the minimizing point found in the previous step;
12. increase the Newton iteration counter : I = I+1 ;
13. reiterate steps 5 through 12 until the values

    $$\underline{X}(I) = (C1,C2,C3,U)$$

    converge within acceptable tolerances;
14. calcualte the estimate of S^2 using (4.17).

This method shall hereafter be referred to as the 'Maximum Likelihood Spherical Method', or M.L.S. for short. As will be seen from the evaluations made in Chapter V, the M.L.S. method is apparently the only alternative to consistently rival the performance of the currently used NAVY_A method. It has the additional advantage that it estimates the error present in the time data values.

F. A COMPUTATIONAL EXAMPLE

This latest method, M.L.S., is now applied to the same example considered in Chapters II and III. For a quick comparison, Table IV lists the results of using all the methods. The error vector lengths are the distances of each position estimate from the EXACT apparent position.

Since this is only one example, this table is not presented for the purpose of any broad conclusions. However it is of interest to note that in this example

1. the M.L.S. method outperforms all others, including the NAVY_A method; and
2. in many ways the original NAVY method outperforms the adjusted NAVY_A method.

G. VARIANCE ESTIMATION

In the computational example, the time data values used were exact since the methods of Appendix A could be used with the known linear velocity profile. Therefore the appropriate value for variance in the time values would be zero. For this error free example, the estimates shown in Table V were obtained for the standard deviations of timing noise, using the two maximum likelihood methods.

In the case of this one example, the spherical assumption is apparently an improvement over the planar, since the M.L.S. estimate of error is only 4% of the M.L.P. estimate.

TABLE IV

Single Example Comparison of All Methods

Method	Transit Time T (secs)	Elev. Angle A (degs)	Length of Error Vector
NAVY	0.67526969	15.26459	0.4128
NAVY_A	0.67527027	15.26461	0.4840
L.S.	0.67529319	15.22798	3.739
L.S.C.	0.67527149	15.26372	0.522
M.L.P.	0.67529007	15.10437	13.640
M.L.S.	0.67527049	15.26677	0.4117
EXACT	0.67527043	15.27002	

	Apparent Position Estimate
NAVY	(1005.957 , 3017.874 , 868.143)
NAVY_A	(1006.087 , 3018.259 , 868.255)
L.S.	(1003.809 , 3019.751 , 866.126)
L.S.C.	(1006.089 , 3018.266 , 868.235)
M.L.P.	(997.722 , 3023.685 , 859.355)
M.L.S.	(1006.195 , 3018.190 , 868.375)
EXACT	(1005.966 , 3017.899 , 868.555)

That is regarded as an improvement because the correct answer is zero. The higher M.L.P. estimate is indicative of the inflation due to the planar assumption.

The tracking range which provided data for this study records time values to seven decimal places. Therefore the standard deviation estimated by the M.L.S. method is of particular interest, since it indicates errors in the seventh decimal place even when there is no error present. Since the data was actually error free in this example, the

TABLE V

Maximum Likelihood Error Estimates
for the Example Problem

Model	Method	Est. Std. Deviation
Planar	M.L.P.	5.517 E-6 secs.
Spherical	M.L.S.	2.191 E-7 secs.

estimate is a measure of the variation induced by the spherical wavefront assumption. A broader discussion of error estimation will be undertaken in Chapter VI.

V. EVALUATIONS OF MODELS

A. GENERAL

There are many sources of errors in the overall hydrophonic tracking problem. These include, among others:

1. errors in estimation of the sound velocity profile;
2. inhomogeneity of the velocity profile over time and horizontal displacement; and
3. possible errors in measuring the positions, and the angles of tilt and rotation for the hydrophonic arrays.

This study focuses on those errors which occur during the computations preceding the ray tracing procedure. To evaluate the performance of the methods developed in Chapters II, III and IV, it is necessary to control strictly the ray tracing procedure. Only in that way can the differences found between methods be attributed to the differences between models, and not to any source of error outside those methods.

It was originally hoped that the various methods might be compared by applying them to real tracking data. However it was found that the overall tracking problem had too many large sources of error to allow the methods to demonstrate any differences. Therefore the methods were compared under a more tightly controlled simulated environment.

B. SIMULATION SCENARIO

Two different simulations are used to compare the six different methods. Both use a basic scenario similar to that of the computational example explored in Chapters II, III and IV. That example assumed that:

1. the velocity versus depth relationship is linear, and given exactly by:

$$V = 4840.7 + 0.03314 \bullet \text{DEPTH} ;$$

2. the acoustic center of each array is at a depth of 1300 feet;
3. the hydrophone arrays are all level, and their X, Y and Z arms are parallel to the respective coordinate axes of the tracking range.

Under these circumstances the methods set forth in Appendix A can be used to compute the exact values for the hydrophone times, ray transit time and elevation angle for a sound wave emanating from a source at any specified location. Therefore when the methods are applied to those exact times, the resulting estimated positions can be compared to the known true position.

C. SIMULATED ERROR FOR TIMING DATA

The models developed in this study were designed to improve position estimation, especially in the presence of errors in the timing data. Lacking any better model at this time for timing errors, the simulated environment includes an assumption of Gaussian errors for the hydrophone times. Therefore realistic timing data can be simulated by adding to each exact time value a random quantity of normally distributed error. The mean of the error is assumed to be zero. The variance was estimated from real tracking data, using the variance estimating property of the M.L.S. model. The data from one tracking run was used, involving six hydrophone arrays and 733 position estimates (see Chapter VI for data selection details). Each position from the tracking run produces one estimate of the variance. The variance value chosen for use in the simulations was the median of the 733 variance estimates produced by the M.L.S. method. That value was

$$S^2 = 9.1204\ E{-}12\ \ \text{secs}^2 .$$

That is the same as a standarad deviation of

$$S = 3.02\ E{-}6\ \ \text{secs} .$$

Each of the two types of simulations was run four separate times. Each run was done with a different specified variation. Those four distinct error conditions are delineated in Table VI .

TABLE VI

Simulation Error Levels

RUN	LEVEL	VARIANCE	STD. DEVIATION
1	zero	0.0	0.0
2	low	9.12 E-14	3.02 E-7
3	medium	9.12 E-12	3.02 E-6
4	high	9.12 E-10	3.02 E-5

D. SINGLE ARRAY SIMULATION

In the first simulation, the intent was to compare the methods pairwise, so as to determine which method is more likely to produce the more accurate estimate of a given sound source position. One thousand positions were chosen at random. Each position was 3000 feet from the array. The positions were uniformly spread over the surface of a sphere of radius 3000 feet, centered at the array, truncated above by the water surface (depth 0) and below by the depth of the array (1300 feet). The methods set forth in Appendix C were used to assure that the random positions selected were

uniformly distributed over the surface area of the truncated hemisphere.

Each of the 1000 randomly chosen source positions was then processed as follows:

1. calculate the exact hydrophone times, using the methods of Appendix A;
2. add to each of the four exact times a random value of error at the specified level (zero, low, medium or high);
3. apply each of the six methods to the hydrophone times, generating six different apparent positions;
4. apply the ray tracing procedure to each of the six positions, using layers that are 25 feet thick, and utilizing the known linear velocity profile, thereby producing six different estimates of the sound source location;
5. compare the six different estimates pairwise to see which method in each pair produced the estimate closest to the true sound source location.

The comparison being made is that one method is considered preferrable to the other if it more frequently produces the more accurate estimate.

The layer thickness of 25 feet was selected order to simulate the actual procedure at the tracking range which provided data for this study. However, as discussed in Chapter I, when the velocity profile is smooth and known exactly, the process is very robust with respect to the thickness used. The thickness values 1, 10, and 25 were all tried, with virtually no changes in any of the comparisons between methods.

E. SINGLE ARRAY SIMULATION RESULTS

Tables VII and VIII contain the results of the single array simulation. Each tabled entry represents the fraction of time that the method of that row produced a better estimate than the method of that column. For example, in Table VII, the L.S. method outperformed the M.L.S. method in only 5.8% of the 1000 trials with low error values.

In a one tailed test that one method is better than another, these binomial proportions are significant at the 0.05 level if they exceed 0.526. Symmetrically, one method is significantly worse than another at the 0.05 level if the proportion is less than 0.474. For the 0.01 level the corresponding critical values are 0.537 and 0.463 respectively.

The results indicate that:

1. as previously claimed, the NAVY_A method usually outperforms the unadjusted NAVY method; of particular interest is the case of zero error which actually compares the relative ability of each method to produce the exact answer when given the exact times; in those cases the NAVY_A method does extremely well against the NAVY method;
2. under all error conditions the most successful performer is the M.L.S. method, since it always has a favorable (greater than 0.5) comparison fraction against all other methods; the M.L.S. fractions vary little over the four error levels;
3. under all error conditions, the worst performer is always the M.L.P. method;
4. spherical methods consistently outperform planar methods; and
5. increased error levels tend to lessen the distinction between methods; in the zero error case comparisons

TABLE VII

Single Array Simulation Results for Lower Error Levels

ERROR LEVEL : ZERO

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.011	.950	.676	.987	.420
NAVY_A	.989		.950	.728	.987	.434
L.S.	.050	.050		.050	.987	.057
L.S.C.	.324	.272	.950		.987	.346
M.L.P.	.013	.013	.013	.013		.013
M.L.S.	.580	.566	.943	.654	.987	

ERROR LEVEL : LOW

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.165	.952	.648	.987	.396
NAVY_A	.835		.952	.693	.987	.405
L.S.	.048	.048		.048	.987	.058
L.S.C.	.352	.307	.952		.987	.369
M.L.P.	.013	.013	.013	.013		.013
M.L.S.	.604	.595	.942	.631	.987	

are in the interval (0.01,0.99), while in the high error case that interval is narrowed considerably to (0.37,0.63).

F. DOUBLE ARRAY SIMULATION

In the second simulation, the intent again was to compare the methods pairwise, this time determining which method is more likely to produce positions which agree more closely in the two array crossover problem. This is not the

TABLE VIII

Single Array Simulation Results for Higher Error Levels

ERROR LEVEL : MEDIUM

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.464	.819	.559	.987	.435
NAVY_A	.546		.821	.566	.987	.437
L.S.	.181	.179		.181	.971	.177
L.S.C.	.441	.434	.819		.987	.450
M.L.P.	.013	.013	.029	.013		.013
M.L.S.	.565	.563	.823	.550	.987	

ERROR LEVEL : HIGH

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.486	.544	.439	.562	.431
NAVY_A	.514		.546	.516	.559	.431
L.S.	.456	.454		.457	.557	.400
L.S.C.	.561	.484	.543		.562	.427
M.L.P.	.438	.441	.443	.438		.373
M.L.S.	.569	.569	.600	.573	.627	

same question as that addressed by the single array simulation. The two estimates produced by any one method may be very close to each other, and yet be far away from the true position.

For the double array scenario two arrays are used, separated by 7500 feet, both at depths of 1300 feet. The arms of each array are parallel to the corresponding coordinate system axes. Once again 1000 positions were randomly generated in a uniform manner, this time over a 3 dimensional box

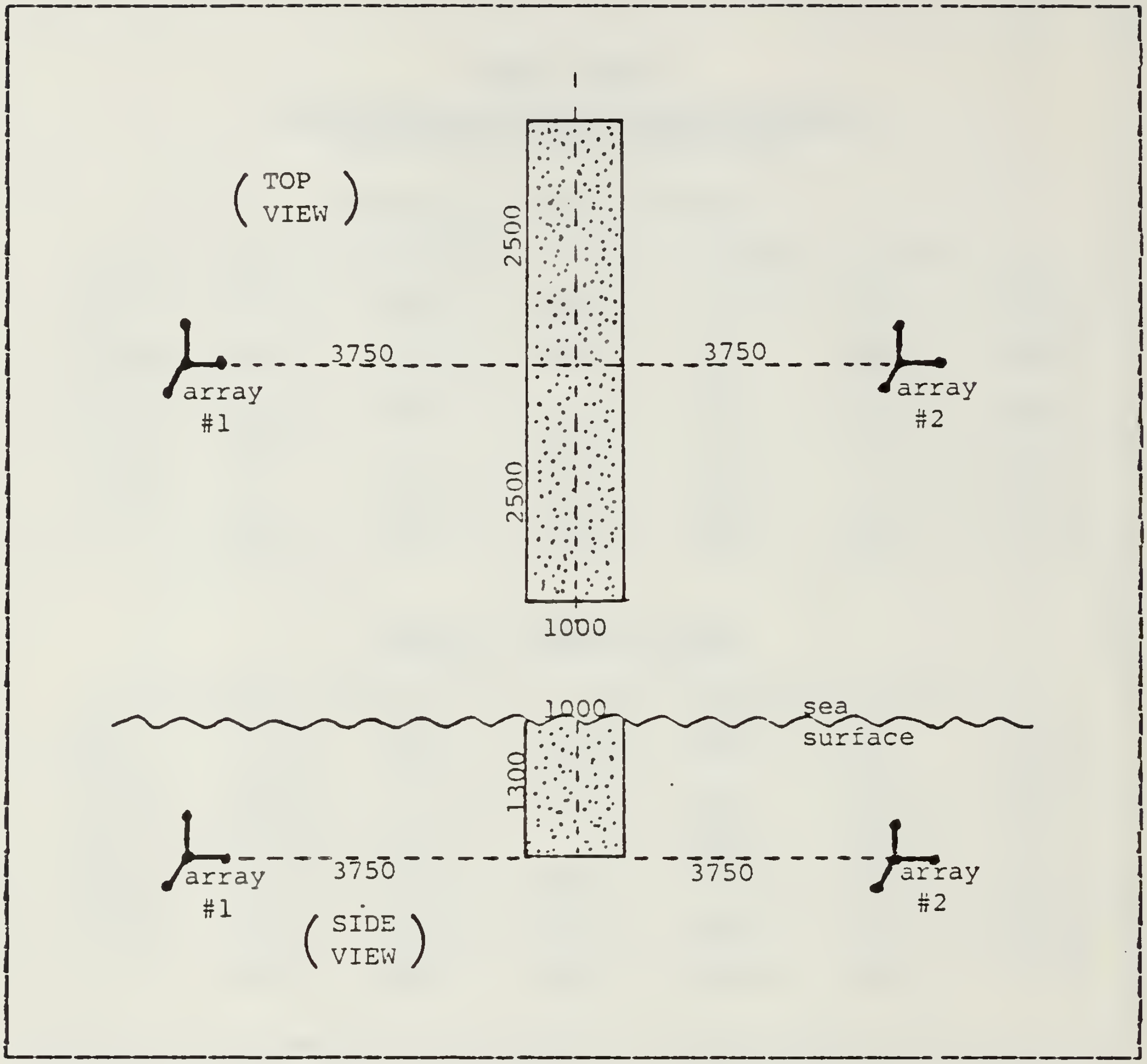


Figure 5.1 Double Array Simulation Configuration.

running crossways between the two arrays. The box is 1300 feet deep, 5000 feet long and 1000 feet across (see figure 5.1).

Each of the 1000 randomly chosen sound source positions in the two array simulation were processed as follows:

1. calculate the exact hydrophone times for the first array using the methods set forth in Appendix A;

2. add to the exact times some random error at the specified level;
3. Repeat steps 1 and 2 for the second array, using a new set of random error values;
4. apply each of the six methods to the time values from both arrays, producing six different pairs of apparent positions;
5. apply the ray tracing procedure to both apparent positions in each of the six pairs, utilizing the known linear velocity profile, producing six pairs of estimated sound source positions;
6. for each of the six pairs of positions, calculate the distance between the two positions in the pair;
7. make pairwise comparisons of the six different distances, to see which method in each pair exhibits the closest agreement between its two position estimates.

This time the comparison being made is that one method is considered better than another if the positions it produces agree more closely more often than those of the other method.

G. DOUBLE ARRAY SIMULATION RESULTS

Tables IX and X contain the results of the double array simulation. Each tabled entry represents the fraction of time that the method of that row produced a pair of estimates which were in closer agreement than the estimates produced by the method of that column. Significance criteria for these proportions are the same as for the single array simulation.

These results are similar to those of the single array simulation, because they indicate that:

TABLE IX

Double Array Simulation Results for Lower Error Levels

ERROR LEVEL : ZERO

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.366	.989	.726	.968	.212
NAVY_A	.634		.989	.734	.968	.212
L.S.	.011	.011		.011	.555	.014
L.S.C.	.274	.266	.989		.968	.193
M.L.P.	.032	.032	.445	.032		.033
M.L.S.	.788	.788	.986	.807	.967	

ERROR LEVEL : LOW

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.448	.989	.661	.952	.310
NAVY_A	.552		.989	.684	.952	.312
L.S.	.011	.011		.011	.560	.014
L.S.C.	.339	.316	.989		.952	.254
M.L.P.	.048	.048	.440	.048		.038
M.L.S.	.690	.688	.986	.746	.962	

1. the NAVY_A method outperforms the original unadjusted NAVY method, although the difference is not significant in the higher error level cases;
2. the most successful performer is consistently the M.L.S. method;
3. spherical methods almost always outperform planar methods;
4. increased error levels tend to lessen the distinction between methods.

TABLE X

Double Array Simulation Results for Higher Error Levels

ERROR LEVEL : MEDIUM

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.498	.841	.513	.808	.443
NAVY_A	.502		.842	.542	.809	.440
L.S.	.159	.158		.159	.525	.160
L.S.C.	.487	.458	.841		.808	.433
M.L.P.	.192	.191	.475	.192		.144
M.L.S.	.557	.560	.840	.567	.856	

ERROR LEVEL : HIGH

	NAVY	NAVY_A	L.S.	L.S.C.	M.L.P.	M.L.S.
NAVY		.487	.556	.535	.479	.442
NAVY_A	.513		.558	.499	.481	.440
L.S.	.444	.442		.445	.451	.422
L.S.C.	.465	.501	.555		.480	.438
M.L.P.	.521	.519	.549	.520		.428
M.L.S.	.558	.560	.578	.562	.572	

There are however some indications from these results which are different from those of the single array simulation, such as:

1. the worst performer was consistently the L.S. method, rather than the M.L.P. method;
2. in the high error case the M.L.P. method is equal to, or even marginally better than, every other method except the M.L.S. method;

3. the performance of the M.L.S. method is noticeably better under error free conditions.

The last two inferences are perhaps the most interesting. The maximum likelihood approach was intended to estimate and account for Gaussian errors in the timing data values. Hence it is really not very surprising that the M.L.S. method does well with error prone data. But it is interesting to note that the M.L.P. method also does well under high error levels, even though it probably suffers from a bias due to the planar assumption.

Of even greater interest is that the M.L.S. method seems to be at its best when compared to other methods under error free conditions. This result was unexpected, and indicates that the M.L.S. method not only handles timing errors well, as was intended, but apparently also does an even better job of approximating the elusive exact solution to the original four spherical equations of (4.1) and (2.1) when the exact time values are available.

H. LIMITATIONS ON INTERPRETATION OF RESULTS

The results of both simulations seem to imply that the M.L.S. method outperforms the currently used NAVY_A method on any randomly chosen sound source position, with or without timing errors. These are encouraging results. Nevertheless the reader is cautioned that these tests are just simulations, and like all simulations, must make assumptions which cannot fully reflect the reality of actual hydrophonic tracking conditions. The most important assumptions made for these simulations are:

1. the sound velocity profile is known exactly, and is linear;
2. the errors in the timing values from any hydrophone are normally distributed with mean zero, and are independent of the noise in any other hydrophone; and

3. the parameters of the simulation were fixed; for example the single array simulation used a fixed range of 3000 feet, and both simulations used arrays at 1300 feet of depth, with fixed orientations to the range coordinate system.

The first assumption is probably of little consequence. It not only greatly facilitates computations, but also helps to isolate the initial angle and time estimation problem from the unrelated errors involved in the velocity profile estimation procedure.

The second assumption is somewhat more troublesome. Errors may not be Gaussian at all, or if they are, the mean may not be zero. Unfortunately each position estimation involved only four equations, and therefore did not allow for estimation of more than four paramenters. Therefore the error mean, being a fifth parameter, could not be estimated. Also the errors of any one hydrophone may very well not be independent of the errors of the other three phones on that array. Fortunately these concerns are offset somewhat by the results of the double array simulation, wherein it was found that the M.L.S. method was at its best when there was no noise at all.

Also the error type and level may depend on other factors, such as the target's range, elevation and azimuth angles from the array. This highlights the concerns of the third assumption. There is considerable room here for future work concerning the dependency of results on such complicating factors.

Lastly it should be pointed out that the simulations make comparisons only on the binomial basis of better versus worse in 1000 trials. The magnitudes of the actual differences are ignored. It is possible, though perhaps unlikely, that while one method marginally outperforms a second method in most trials, in all the remaining trials the first method

is much worse than the second. There is also room here for further work.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. ESTIMATION OF TIMING DATA VARIATION

One of the primary purposes of this study was to estimate the amount of variability in the timing data being recorded during actual tracking runs. This problem was addressed by the M.L.P. and M.L.S. maximum likelihood models. The M.L.P. model, as previously discussed, suffered from a bias due to the planar assumption, was the poorest estimator of positions among all the models, and produced an inflated variance estimate. Therefore the spherical model M.L.S. is used to estimate the data variance.

The variability that is measured by the M.L.S. model is made up of three components. First there is the variance induced by the spherical assumption. Then there is the variance caused by the seven decimal accuracy used when recording the data. Finally there are the errors inherent in the physical process, due to such factors as hydrophone variability or malfunction, local distortions of the sound wave, and inexactness of the water column which estimates the speed of sound profile. The last two sources of error together make up the variability that is involved in the time values which are ultimately used in position estimation, and is therefore the variation that is to be estimated.

If it is assumed that the variability induced by the spherical assumption is independent of the data variability, then the M.L.S. variance estimate is the sum of those two variances, or

$$\sigma^2_{mls} = \sigma^2_{sph} + \sigma^2_{time}$$

Therefore the data variance can be estimated by first estimating the variability induced by the spherical assumption, and then subtracting it from the M.L.S. estimate of the variation.

The M.L.S. estimate was obtained by applying the M.L.S. method to the data from a tracking run at the Nanocse torpedo tracking range on May 6, 1980. That run involved position estimation by several different hydrophone arrays. The run made several thousand position estimates, 733 of which were at depths of 100 feet or more and involved targets not more than 4700 feet from the sensing array. The depth limitation was imposed to avoid the excessive complications caused by the radical changes in the velocity profile above that depth (see figure 2.1). The maximum range limitation imitates the data validation procedure at the tracking range, where positions farther than 4700 feet from the array are discarded.

The tracking data yielded the following range of estimates for the standard deviation of the data noise, using the M.L.S. model.

MAXIMUM VALUE	2.89 E-5	secs.
0.95 QUANTILE	1.18 E-5	secs.
MEDIAN VALUE	3.02 E-6	secs.
0.05 QUANTILE	4.11 E-7	secs.
MINIMUM VALUE	3.13 E-8	secs.

For an overall estimate of the noise, the median value was used, so that:

$$\sigma^2_{mls} = (3.02 \text{ E-6})^2$$

The variability induced by the spherical assumption was estimated by applying the M.L.S. procedure to perfectly noiseless data in the idealized environment of the single array simulation of Chapter V. This was done for targets at ranges of 1500 to 4500 feet, at 500 feet increments, with

1000 randomely chosen targets at each range. The results are collected in Table XI .

TABLE XI

Inflation of Error Estimates Induced by the Spherical Model

Standard Deviation Estimates from Exact Time Values with Known Linear Velocity Profile

RANGE	MINIMUM	Q(.05)	MEDIAN	Q(.95)	MAXIMUM
1500	2.72E-10	2.84E-8	2.08E-7	3.13E-7	3.29E-7
2000	2.12E-9	5.88E-8	2.26E-7	3.14E-7	3.30E-7
2500	3.90E-8	1.09E-7	2.34E-7	3.14E-7	3.31E-7
3000	8.35E-8	1.50E-7	2.37E-7	3.15E-7	3.24E-7
3500	1.21E-7	1.59E-7	2.37E-7	3.13E-7	3.24E-7
4000	1.47E-7	1.65E-7	2.39E-7	3.16E-7	3.25E-7
4500	1.46E-7	1.69E-7	2.42E-7	3.14E-7	3.25E-7

Table XI shows that the median and maximum inflation values are reasonably independent of target range. The minimum values vary somewhat, but only for range values below 3000 feet. This represents a very stable situation overall. Therefore the inflation due to the spherical assumption is estimated by the median value at a range of 3000 feet, namely:

$$\sigma^2_{sph} = (\ 2.37\ E\text{-}7\ )^2\ .$$

Combining these two estimates, the variance estimated for the timing data is

$$\sigma^2_{time} = \sigma^2_{mls} - \sigma^2_{sph}$$
$$= (3.02\ E\text{-}6)^2 - (2.37\ E\text{-}7)^2$$
$$= (3.011\ E\text{-}6)^2 \ .$$

As can be seen, the error induced by the spherical model is less than 10% of the M.L.S. error estimate. Therefore when it is accounted for by subtraction from the M.L.S. variation estimate, the final variance estimate changes little.

The estimated value indicates a standard error in the 6th decimal place. With a typical speed of sound value of 4880 feet per second, this represents a position differential of about

$$4880 \bullet 3.011E\text{-}6 = 0.015 \text{ feet} \ .$$

This estimate is quite low, indicating that the time values being recorded are sufficiently accurate.

There is considerable opportunity for additional work determining the relationship, if any, between the time variance and other factors such as angles of elevation and azimuth of the target from the array.

There is also the problem that the time variation is likely to be array dependent. For example, consider figure 6.1 , wherein the standard error estimates from the actual tracking run are plotted versus the range of the target. The plot does not indicate that there is any simple relationship between range and error level. However the plot does show a bunching pattern. When the error estimates are plotted separately for each array, then the bunching pattern becomes clearly associated with the individual arrays. Consider figures 6.2 and 6.3, where the separate plots have been made for four different arrays. It is still not clear from these plots whether the principle effect is due to the individual arrays, or the ranges of the targets. However some level of array dependency seems likely, indicating a need for additional investigation.

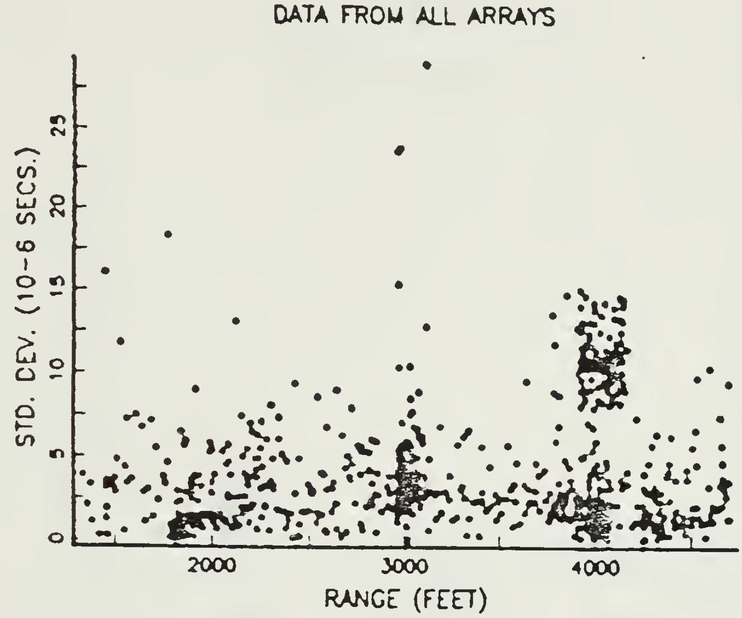


Figure 6.1 Error Estimation Versus Range of Target.

B. CHOICE OF METHOD

Clearly all indications are that the planar wavefront models, L.S. and M.L.P. are not candidates for use as position estimators. Furthermore the hybrid model L.S.C. is an interesting improvement, but never really performs well enough compared to the M.L.S. and NAVY models.

The original NAVY model is usually outperformed by the adjusted NAVY_A method. However the differences are not always significant.

The spherical model M.L.S., on the other hand, consistently outperformed all other methods during the simulated evaluations. It would seem that M.L.S. is the model of choice. It does the best job of handling normally distributed errors in the data. But that is not the strongest

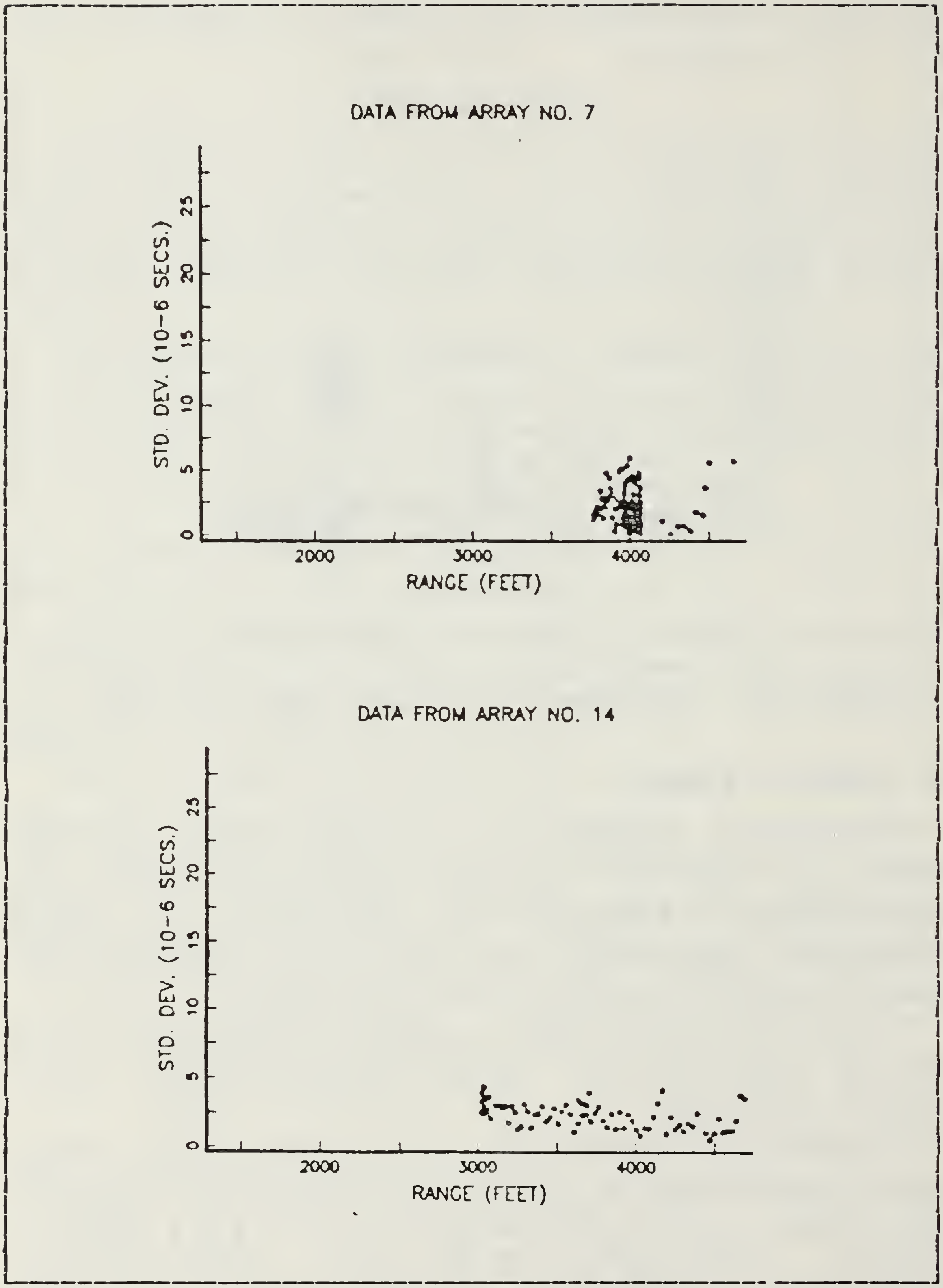


Figure 6.2 Error Estimation for Arrays 7 and 14.

Figure 6.3 Error Estimation for Arrays 56 and 57.

argument for its use. A more important, and surprising, argument in its favor is that when the exact, error free times are used, the M.L.S. apparent position estimate will usually produce the most accurate estimate of the true apparent position. This is the desired overall result, so that the sensitive ray tracing procedure will be affected minimally by apparent position estimation errors.

There are nevertheless several notes of caution which should be considered before embracing the M.L.S. method wholeheartedly. The first caution has been stressed before, namely that these conclusions were arrived at under the idealized conditions of the simulations scenarios. The second caution, also previously stressed, is that the actual magnitudes of the differences between position estimates has been ignored. It is conceivable that while one method always produces a better estimate than another, the difference between any two position estimates is acceptably small.

Lastly there is the caution that the M.L.S. model involves a complicated iterative procedure which uses considerable computer time. It is probably too slow a procedure for use with 'real time' analysis during the execution of tracking runs. For real time tracking the NAVY_A method currently in use is probably preferrable due to its simple computations.

However, for post run analysis, and also possibly for calibration of the hydrophone arrays, the M.L.S method is recommended as being a more exact and more robist position estimator than those methods currently in use.

C. RECOMMENDATIONS FOR FUTURE INQUIRY

Several recommendations have already been made for work needed to estimate the effect of suitable independent variables on both timing errors and the bias in certain methods.

In addition there exist at least two other areas for possibly fruitful investigation.

The first area concerns the interplay between methods. Specifically, the binomial comparisons of Chapter V show that even the worst methods are better than each of the other methods at least part of the time. Hence it is possible that the best method overall would be a suitable combination of methods, wherein each method is used where it is most effective. For example, even though the M.L.S. method has been indicated as the best method for any randomly selected position, it may be consistently outperformed by another method under certain circumstances, such as extremely high or low elevation angles.

The second area for possible work addresses the question of how to next improve upon the existing models. It is herein suggested that the next improvement in modelling would be a method which is based on a linear velocity assumption. As figure 2.1 demonstrates, a linear velocity profile is a reasonable approximation for most depths. This would be the next logical step above the constant velocity assumption which is associated with the spherical models. Most of the mathematical basis for such a model in contained in Appendix A. Possibly a suitable set of equations could be developed involving the hydrophone times and reflecting the linear velocity assumption. If so, the least squares or maximum likelihood techniques might provide useful results.

APPENDIX A

LINEAR VELOCITY PROFILE THEORY

All computations in this study are made under the assumption that the sound velocity is directly related to depth in a linear manner, and is known exactly. Under these circumstances many closed form results, not otherwise available, can be obtained and used in those computations.

Suppose that the velocity profile is given by

$$V = V0 + V1 * z$$

where V0 and V1 are known constants, and z is the depth variable, measured down from the water's surface. In this case it is known [Ref. 4] that the path of a sound ray from a signal source to a hydrophone will have the shape of an arc of a circle. The center of that circle will be somewhere above the surface of the water. The vertical placement of that center is determined by the value of z at which the speed of sound equals zero (see figure A.1). Although that depth is negative and is not really a depth at all, it nevertheless has geometric meaning.

Consider the vertical plane containing the circle center, the sound source and the hydrophone. Let h be the variable which measures the horizontal position in that plane. Let (h,z)=(a1,a2) be the position of the hydrophone, and let (h,z)=(p1,p2) be the position of the sound source. Then C2 given by (A.1) is the Z coordinate of the circle center.

$$C_2 = \frac{-V_0}{V_1} \tag{A.1}$$

What must be found is C1, the h coordinate of the circle center, and r, the radius of the circle. To solve for these values, the equation of the circle is used, evaluated at the the two known locations, yielding (A.2).

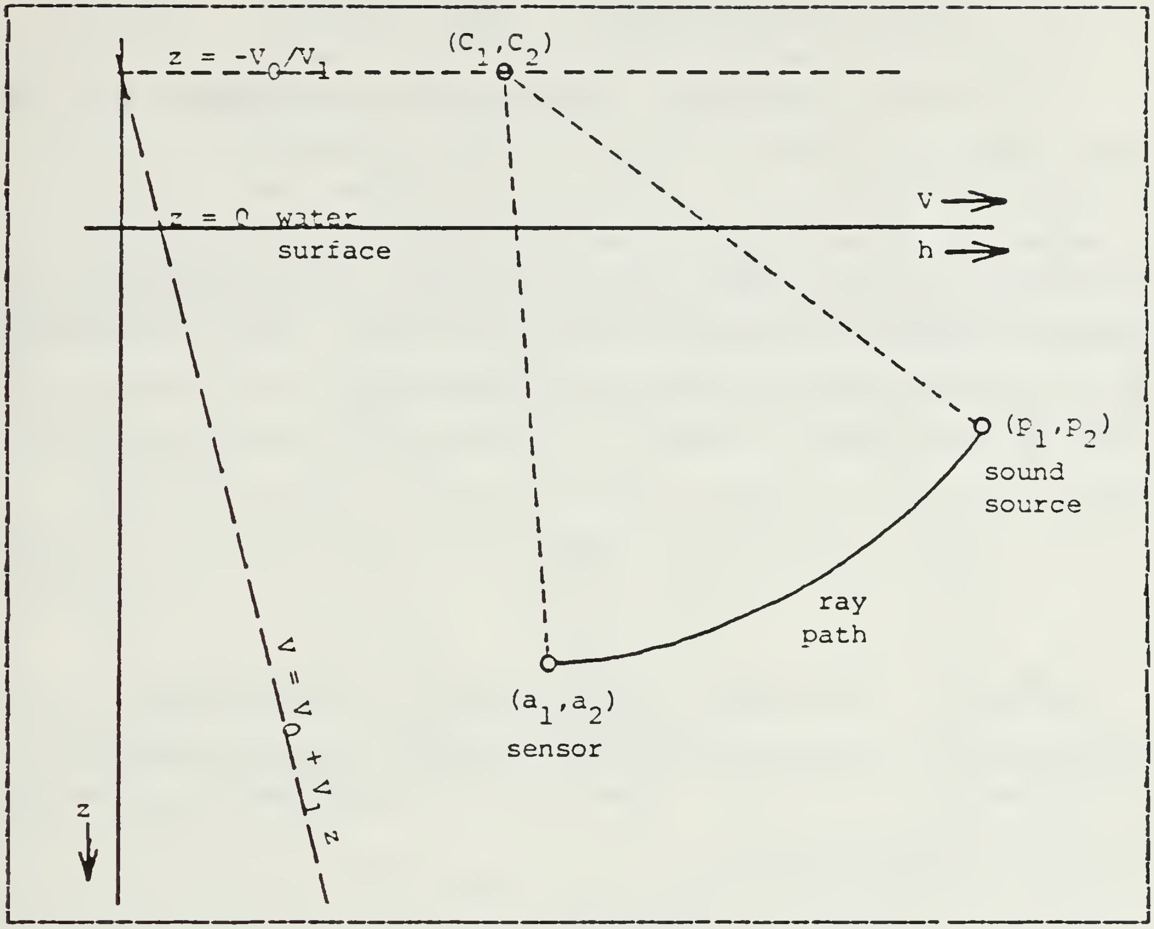


Figure A.1 Circular Ray Paths for a Linear Velocity Profile.

$$\begin{aligned} (a_1 - C_1)^2 + (a_2 - C_2)^2 &= r^2 \\ (p_1 - C_1)^2 + (p_2 - C_2)^2 &= r^2 \end{aligned} \tag{A.2}$$

The left hand sides of the two equations of (A.2) can be equated, and solved for C1, leading to (A.3).

$$C_1 = \frac{(p_1 + a_1)}{2} + \frac{(p_2 - a_2)}{2(p_1 - a_1)} (P_2 + a_2 - 2C_2) \tag{A.3}$$

When this value of C1 is substituted into the first equation of (A.2), then the result is given by (A.4).

$$r = \sqrt{(a_1 - c_1)^2 + (a_2 - c_2)^2} \tag{A.4}$$

The circular arclength between the hydrophone and the sound source is easily computed. Unfortunately the velocity of sound does not stay constant along that arc. Therefore in order to determine the amount of time (T) required for the sound ray to travel the ray path, the effect of the velocity must be integrated along the arc. This is done by (A.5), where S is the arclength between the two points, and V(s) is the speed of sound as a function of position on the arc.

$$T = \int_0^S \frac{ds}{V(s)} \tag{A.5}$$

In (A.5) the sound source position corresponds to an arclength of s = 0, and the hydrophone position correponds to arclength s = S. In [Ref. 4] this integral is shown to be equivalent to (A.6).

$$T = \frac{-1}{V_1} \int_{A_0}^{A_1} \frac{da}{\cos(a)} \tag{A.6}$$

In (A.6) A0 is the angle of elevation of the ray path at the sound source, and A1 is the elevation angle at the hydrophone. The antiderivative in (A.7) can be used to solve (A.6), leading to the ray path transit time expression in (A.8).

$$\int \frac{da}{\cos(a)} = \ln\left(\frac{1 + \sin(a)}{\cos(a)}\right) \tag{A.7}$$

$$T = \frac{1}{V_1} \ln\left[\left(\frac{\cos(A_1)}{\cos(A_0)}\right)\left(\frac{1 + \sin(A_0)}{1 + \sin(A_1)}\right)\right] \tag{A.8}$$

If the elevation angle at any point along the arc is denoted by A, then (A.9) relates the angle to the derivative of z with respect to h along the ray path.

$$\tan(A) = \frac{dz}{dh} \tag{A.9}$$

Implicit differentiation of the equation of the circle yields (A.10) as another expression for the same derivative.

$$\frac{dz}{dh} = \frac{h - C_1}{z - C_2} \tag{A.10}$$

Equating these two derivatives leads to (A.11) which relates the elevation angle and the position (h,z) on the arc.

$$\tan(A) = \frac{h - C_1}{z - C_2} \tag{A.11}$$

From (A.11) a simple geometric argument produces the equations in (A.12).

$$\cos(A) = \frac{z - C_2}{r} \qquad \sin(A) = \frac{h - C_1}{r} \tag{A.12}$$

First let A, h and z be equal to (A1,a1,a2) in (A.12), and then let them equal (A0,p1,p2). Then substitute both expressions into (A.8). The result is (A.13), the desired expression for the ray path transit time in terms of the positions of the sound source and the hydrophone.

$$T = \frac{1}{V_1} \ln\left[\left(\frac{a_2 - C_2}{p_2 - C_2}\right)\left(\frac{r + p_1 - C_1}{r + a_1 - C_1}\right)\right] \tag{A.13}$$

To summarize, if a ray path is to terminate at the three dimensional position (X1,Y1,Z1), and the sound source is at (X2,Y2,Z2), then perform the following steps in order to calculate the exact ray path time and elevation angle that would correspond to a linear velocity profile:

1. use (A.14) to translate the three dimensional positions to the two dimensional coordinates of the previously described vertical plane, with the origin at the water surface directly above the end of the ray path;

$$p_1 = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} \qquad (A.14)$$

$$p_2 = Z_1 \qquad a_1 = 0 \qquad a_2 = Z_2$$

2. calculate C2, C1 and r using (A.1), (A.3), and (A.4);
3. calculate the exact ray path transit time T using (A.13); and
4. calculate the exact elevation angle A at the hydrophone, using (A.15).

$$A = \arccos\left(\frac{a_2 - C_2}{r}\right) \qquad (A.15)$$

APPENDIX B

PARTIAL DERIVATIVE FORMULAE FOR NEWTON'S METHOD

The central tool used by Newton's Method in the development of the M.L.S. model in Chapter IV is the matrix GP. It is the matrix of first order derivatives of the error expressions g1, g2, g3 and g4. Those derivatives are set forth in this appendix.

If $\underline{X}$ = (C1,C2,C3,U), then the error functions are defined by (B.1),

$$g_i(\underline{X}) = C_i - \frac{T_i - \sqrt{K_i}}{\sqrt{K_i}\ M} \qquad i = 1,2,3 \qquad (B.1)$$

$$g_4(\underline{X}) = U - \frac{T_4 - D\ M\ /\ V}{1 - N}$$

where the values Ki, M and N are the functions given by (B.2).

$$K_i = U^2 - (\ 2UDC_i/V\ ) + \frac{D^2}{V^2} \qquad (B.2)$$

$$N = \sum_{j=1}^{3} C_j\left(\frac{T_j - \sqrt{K_j}}{\sqrt{K_j}}\right)$$

$$M = \sum_{j=1}^{3} \frac{T_j - \sqrt{K_j}}{\sqrt{K_j}}$$

Recall that GP is the matrix shown in (B.3).

$$GP\ (C_1,C_2,C_3,U) = \begin{bmatrix} \frac{\partial g_1}{\partial c_1} & \frac{\partial g_1}{\partial c_2} & \frac{\partial g_1}{\partial c_3} & \frac{\partial g_1}{\partial c_4} \\ \frac{\partial g_2}{\partial c_1} & \frac{\partial g_2}{\partial c_2} & \frac{\partial g_2}{\partial c_3} & \frac{\partial g_2}{\partial c_4} \\ \frac{\partial g_3}{\partial c_1} & \frac{\partial g_3}{\partial c_2} & \frac{\partial g_3}{\partial c_3} & \frac{\partial g_3}{\partial c_4} \\ \frac{\partial g_4}{\partial c_1} & \frac{\partial g_4}{\partial c_2} & \frac{\partial g_4}{\partial c_3} & \frac{\partial g_4}{\partial c_4} \end{bmatrix} \qquad (B.3)$$

Then given the definitions above, the following derivatives are the result of straight forward, though tedious, differentiation, and are offerred without detailed proof.

LEMMA 1

$$\frac{\partial M}{\partial C_i} = \frac{V\ K_i\ (T_i - K_i) + C_i T_i U\ D}{V\ K_i \sqrt{K_i}} \qquad i = 1,2,3 \qquad (B.4)$$

LEMMA 2

$$\frac{\partial M}{\partial U} = \sum_{j=1}^{3} \frac{C_j T_j\ (D\ C_j - V\ U)}{V\ K_j \sqrt{K_j}} \qquad (B.5)$$

LEMMA 3

$$\frac{\partial N}{\partial C_i} = \frac{T_i\ U\ D}{V\ K_i \sqrt{K_i}} \qquad i = 1,2,3 \qquad (B.6)$$

LEMMA 4

$$\frac{\partial N}{\partial U} = \sum_{j=1}^{3} \frac{T_j\ (D\ C_j - V\ U)}{V\ K_j \sqrt{K_j}} \qquad (B.7)$$

FORMULA 1

The first three diagonal elements of GP are the derivatives of each gi with respect to Ci (i=1,2,3) and are given by (B.8).

$$\frac{\partial g_i}{\partial C_i} = .1 - \left[\frac{T_i UDM - VK_i(T_i - \sqrt{K_i})\frac{\partial M}{\partial C_i}}{V\ M^2\ K_i \sqrt{K_i}}\right] \qquad (B.8)$$

FORMULA 2

The off diagonal elements in the first three rows and columns of GP are the derivatives of each gi with respect to Cj, and are given by (B.9).

$$\frac{\partial g_i}{\partial C_j} = \left(\frac{T_i - \sqrt{K_i}}{M^2 \sqrt{K_i}}\right)\frac{\partial M}{\partial C_j} \qquad \begin{array}{l} i = 1,2,3 \\ j = 1,2,3 \\ j \neq i \end{array} \qquad (B.9)$$

FORMULA 3

The derivative of each gi with respect to U is given by (B.10).

$$\frac{\partial g_i}{\partial U} = \frac{T_i M(VU - DC_i) + VK_i(T_i - \sqrt{K_i})\frac{\partial M}{\partial U}}{V\ M^2\ K_i \sqrt{K_i}} \qquad (B.10)$$

FORMULA 4

The first three elements of the last row of GP are the derivatives of g4 with respect to each Ci, and are given by (B.11).

$$\frac{\partial g_4}{\partial C_i} = \frac{\frac{\partial N}{\partial C_i}(VT_4 - DM) - D\frac{\partial M}{\partial C_i}(1-N)}{V(1-N)^2} \qquad (B.11)$$

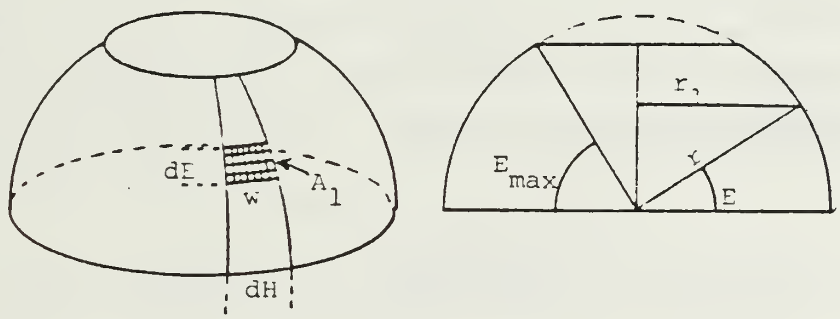
FORMULA 5

The last row, last column of GP is the derivative of $g4$ with respect to U, and is given by (B.12).

$$\frac{\partial g_4}{\partial U} = 1 - \frac{\frac{\partial N}{\partial U}(VT_4 - DM) - D\left(\frac{\partial M}{\partial U}\right)(1 - N)}{V(1 - N)^2} \qquad (B.12)$$

APPENDIX C
UNIFORM SAMPLES ON A TRUNCATED HEMISPHERE

The single array simulation of Chapter V required a random sample of positions in space uniformly distributed over the surface of a truncated hemisphere. The hemisphere is to be of radius r (3000 feet) about the acoustic center of a hydrophonic array. The truncation of the overall sphere is due to the fact that the upper portion of the hemisphere is above the water surface, and the lower half is below the sea bottom.



$$w = r_1 \, dH \qquad r_1 = r \cos(E)$$

$$w = r \cos(E) \, dH$$

Figure C.1 Hemispherical Geometry.

Let E be the variable denoting angles of elevation above the horizontal. Let H be the variable denoting horizontal azimuth angles about the center of the hemisphere. Consider a small piece of hemispherical surface area bounded by the

elevation angles E1 and (E1+dE), and by the azimuth angles H1 and (H1+dH) (see figure C.1). If W is the horizontal width of that piece, then W is given by (C.1).

$$w = r_1 \, dH = r \cos(E) \, dH \qquad \text{(C.1)}$$

If A1 is the area of that piece, then A1 is approximated by (C.2).

$$A_1 \doteq w \, dE = r \cos(E) \, dH \, dE \qquad \text{(C.2)}$$

Now suppose that (0 < E1 < E2 < Emax) where Emax is the elevation angle of the top of the truncated hemishpere. Then the ratio (A1/A2) of two different areas at elevation angles E1 and E2 is given by (C.3).

$$\frac{A_1}{A_2} = \frac{r \cos(E_1) \, dH \, dE}{r \cos(E_2) \, dH \, dE} = \frac{\cos(E_1)}{\cos(E_2)} \qquad \text{(C.3)}$$

If n1 and n2 are to be the (relative) sample sizes from the two areas A1 and A2 respectively, then uniformity of the sample requires that (C.4) hold.

$$A_1 / n_1 = A_2 / n_2 \qquad \text{(C.4)}$$

The combination of (C.4) and (C.3) implies (C.5).

$$n_2 = n_1 \frac{\cos(E_2)}{\cos(E_1)} \qquad \text{(C.5)}$$

Letting E1 = 0, then the relative sample size n2 for area A2 is given by (C.6), where n is the relative sample size at the base of the hemisphere.

$$n_2 = n \cos(E_2) \qquad \text{(C.6)}$$

Now the differential probability of drawing a random position that has elevation angle E is given by (C.7), where

N is the total sample size to be drawn from the surface on the sector bounded by the azimuth angles H1 and (H1+dH).

$$f_E(E) \doteq \frac{n_E}{N} = \frac{n}{N}\cos(E) \qquad (C.7)$$

Therefore, if K is defined to be the constant ratio n/N, then the corresponding cumulative distribution is given by (C.8).

$$F_E(E) = \int_0^E K\cos(e)\,de = K\sin(E) \qquad (C.8)$$

For the highest elevation angle, Emax, the cumulative distribution function must equal one, so that (C.9) holds.

$$F_E(E_{max}) = 1 = K\sin(E_{max}) \qquad (C.9)$$

As a result, the constant K is determined by (C.10).

$$K = \frac{1}{\sin(E_{max})} \qquad (C.10)$$

Therefore the complete cumulative distribution function is given by (C.11).

$$F_E(E) = \frac{\sin(E)}{\sin(E_{max})} \qquad (C.11)$$

Now the inverse probability transform can be used. If U is a uniform random variable on the interval (0,1), then let E be given by (C.12).

$$E = \arcsin\left(\frac{U}{\sin(E_{max})}\right) \qquad (C.12)$$

Now choose an azimuth angle H randomely and uniformly over the interval $(0,2\pi)$. Then (X,Y,Z) given by

$$X = r\cos(H)\cos(E)$$
$$Y = r\sin(H)\cos(E)$$
$$Z = r\sin(E)$$

is the corresponding position in spherical coordinates. That position is a random position drawn from a population of positions uniformly distributed across the surface of a truncated hemisphere with maximum elevation angle Emax.

APPENDIX D

SINGLE ARRAY SIMULATION COMPUTER PROGRAM

```
C       THIS FORTRAN PROGRAM SELECTS A RANDOM SAMPLE OF
C       SIZE M OF 3-DIMENSIONAL POSITIONS ALL AT A RANGE
C       OF R FEET FROM THE ACOUSTIC CENTER OF A HYDROPHONIC
C       ARRAY.  THE ARMS OF THE ARRAY ARE ASSUMED TO BE
C       ALIGNED WITH THE COORDINATE SYSTEM OF THE RANGE.
C       AND IS IN A POSITION GIVEN BY THE VECTOR A.
C
C       THE SPEED OF SOUND PROFILE IS ASSUMED LINEAR,
C       GIVEN BY           V = VV(1)  +  VV(2) * DEPTH  .
C
C       FOR EACH OF THE POSITIONS SELECTED, EXACT HYDROPHONE
C       TIMES ARE CALCULATED USING THE SUBROUTINE TCOMP, AND
C       RANDOM ERRORS ARE ADDED TO THE EXACT TIMES.  THE
C       ERROR DISTRIBUTION IS NORMAL, WITH MEAN ZERO AND
C       STANDARD DEVIATION SDEV.
C
C       THE TIME VALUES ARE THEN USED TO ESTIMATE AN APPARENT
C       POSITION BY TWO DIFFERENT METHODS, USING APPROPRIATE
C       SUBROUTINES.  THE RESULTING APPARENT POSITIONS ARE
C       THEN PROCESSED BY THE SUBROUTINE TRACE TO OBTAIN
C       THE CORRESPONDING ACTUAL POSITION ESTIMATES.  THE
C       RESULTING POSITION ESTIMATES ARE THEN COMPARED TO THE
C       ORIGINAL TRUE POSITION TO SEE WHICH METHOD PRODUCED
C       THE MOST ACCURATE ESTIMATE.
C
C       THIS IS DONE FOR ALL POSSIBLE PAIRS OF THE
C       SIX METHODS.
C
        INTEGER M,I,J,K,N,METH,METH1,METH2,NYES,NYEST,NNOISE
        DIMENSION REL(1000),RAZ(1000),STUFF(4000)
        DOUBLE PRECISION P(1000,3),A(3),VV(2),T(1000,4)
        DOUBLE PRECISION VAR(1000),TN(1000,4),NOISE,FRACT
        DOUBLE PRECISION TEST(1000),PT(1000,3),DIF(1000)
        DOUBLE PRECISION DIFT(1000),AC(3),TC(1000)
        DOUBLE PRECISION R,Z,SDEV,PI,AZ(1000),EL(1000)
        DOUBLE PRECISION V,FK,PHI,PHIMAX,SEED,PEST(1000,3)
C
C       SET INITIAL VALUES OF SAMPLE SIZE, RANGE,
C       AND ERROR STANDARD DEVIATION
C
        M = 1000
        R = 3000.D0
        SDEV = 3.464D-5
  1     FORMAT(2X,' NOISE STD. DEV.  =  ',F15.10)
        WRITE(6,1) SDEV
        WRITE(6,999)
        SEED = 931947.0D0
        PI = 2.D0*DARCOS(0.D0)
C
C       SET ARRAY POSITION AND LINEAR PROFILE
C       FOR SPEED OF SOUND.
C
        A(1) = 0.D0
        A(2) = 0.D0
        A(3) = 1300.D0
        AC(1) = A(1) + 15.0
        AC(2) = A(2) + 15.0
        AC(3) = A(3) - 15.0
```

```
      VV(1) = 4840.7D0
      VV(2) = 3314.D-5
      V = VV(1)+VV(2)*A(3)
C
C     SET UPPER LIMIT ON ELEVATION ANGLE, IF NECESSARY
C
      IF ( R .GT. A(3) ) GO TO 10
         PHIMAX = 1.2566D0
         GO TO 20
 10   PHIMAX =  DARSIN(A(3)/R)
C
C     GENERATE RANDOM VALUES FOR ANGLES OF
C     ELEVATION AND AZIMUTH.
C
 20   FK = 1.D0/DSIN(PHIMAX)
      CALL GGUBS(SEED,M,RAZ)
      CALL GGUBS(SEED,M,REL)
C
C     CONVERT ANGLES TO 3-D COORDINATES
C
      DO 11 I = 1,M
         EL(I) = REL(I)
         AZ(I) = RAZ(I)
         PHI = DARSIN(EL(I)/FK)
         P(I,1) = R*DCOS(PHI)*DCOS(AZ(I)*PI*2.D0)
         P(I,2) = R*DCOS(PHI)*DSIN(AZ(I)*PI*2.D0)
         P(I,3) = A(3)-R*DSIN(PHI)
 11   CONTINUE
C
C     COMPUTE EXACT HYDROPHONE TIMES UNDER THE LINEAR
C     VELOCITY PROFILE ASSUMPTION.
C
      CALL TCOMP(P,AC,M,VV,TC)
      CALL TCOMP (P,A,M,VV,T)
C
C     GENERATE AND ADD ERRORS TO TIMES.
C
      NNOISE = 4*M
 100  CALL GGNML(SEED,NNOISE,STUFF)
      K = 1
      DO 111 I = 1,M
         DO 122 J = 1,4
            NOISE = STUFF(K)
            NOISE = NOISE*SDEV
            TN(I,J) = T(I,J)+NOISE
            K = K + 1
 122     CONTINUE
 111  CONTINUE
C
 301  FORMAT(2X,'          ( SAMPLE SIZE  =  ',I5,' )')
      WRITE(6,301) M
      WRITE(6,999)
C
C     TWO OUTER LOOPS RUN THROUGH ALL PAIRS OF THE SIX
C     POSSIBLE POSITION ESTIMATING METHODS.
C
      DO 1111 METH1 = 1,5
         KMETH = METH1 + 1
      DO 1222 METH2 = KMETH,6
         IF (METH1 .EQ. METH2) GO TO 1222
C
C
 200  FORMAT(2X,'METHODS USED ARE : ')
      WRITE(6,200)
      METH = METH1
```

```
C
C     CALL SUBROUTINES TO IMPLEMENT THE METHODS
C
 130  IF (METH .NE. 1) GO TO 131
         CALL POSNAV (TN,V,M,PEST,TEST)
 201     FORMAT(2X,'                    NAVY, UNCORRECTED')
         WRITE(6,201)
         GO TO 150
 131  IF (METH .NE. 2) GO TO 132
         CALL POSNVC (TN,V,M,PEST,TEST)
 202     FORMAT(2X,'                    NAVY, CORRECTED')
         WRITE(6,202)
         GO TO 150
 132  IF (METH .NE. 3) GO TO 133
         CALL POSLS (TN,V,M,PEST,TEST)
 203     FORMAT (18X,'LEAST SQUARES, UNCORRECTED')
         WRITE(6,203)
         GO TO 150
 133  IF (METH .NE. 4) GO TO 134
         CALL POSLSC (TN,V,M,PEST,TEST)
 204     FORMAT(18X,'LEAST SQUARES, CORRECTED')
         WRITE(6,204)
         GO TO 150
 134  IF (METH .NE. 5) GO TO 135
         CALL POSMLP (TN,V,M,PEST,TEST,VAR)
 205     FORMAT(18X,'MAX. LIKELIHOOD, PLANAR')
         WRITE(6,205)
         GO TO 150
 135  IF (METH .NE. 6) GO TO 136
         CALL POSMLS (TN,V,M,PEST,TEST,VAR)
 206     FORMAT(18x,'MAX. LIKELIHOOD, SPHERICAL')
         WRITE(6,206)
         GO TO 150
 136  WRITE(6,137) METH1,METH2
 137  FORMAT(2X,'CANT FIND METHODS ',I4,' AND/OR ',I4)
      GO TO 1000
C
C     THE APPARENT POSITION ESTIMATES ARE RELATIVE TO
C     THE ARRAY CENTER, AND MUST BE TRANSLATED TO
C     THE TRACKING RANGE COORDINATE SYSTEM.
C
 150  DO 144 I = 1,M
         PEST(I,1) = PEST(I,1) + A(1)
         PEST(I,2) = PEST(I,2) + A(2)
         PEST(I,3) = A(3) - PEST(I,3)
 144  CONTINUE
C
C     CONVERT APPARENT POSITIONS TO ACTUAL ESTIMATES BY
C     RAY TRACING PROCEDURE.
C
      CALL TRACE (PEST,PT,TEST,A,M,VV)
C
C     CALCULATE DIFFERENCE BETWEEN THE 1ST POSITION
C     ESTIMATE AND THE TRUE POSITION.
C
      IF ( METH .NE. METH1 ) GO TO 160
         DO 155 I = 1,M
            DIF(I) = 0.D0
            DIFT(I) = DABS(TEST(I)-TC(I,4))
            DO 156 J = 1,3
               DIF(I) = DIF(I)+(PT(I,J)-P(I,J))**2
 156        CONTINUE
 155     CONTINUE
```

```
C
          METH = METH2
          GO TO 130
C
C     CALCULATE DIFFERENCE BETWEEN THE 2ND POSITION
C     ESTIMATE AND THE TRUE POSITION, AND COMPARE IT
C     TO THE 1ST POSITION DIFFERENCE.
C
 160  NYES = 0
      NYEST = 0
      DO 166  I = 1,M
         DO 167 J = 1,3
            DIF(I) = DIF(I)-(PT(I,J)-P(I,J))**2
 167     CONTINUE
         DIFT(I) = DIFT(I) - DABS(TEST(I)-TC(I,4))
         IF ( DIFT(I) .GT. 0.D0 ) GO TO 165
            NYEST = NYEST + 1
 165     IF ( DIF(I) .GT. 0.D0 ) GO TO 166
            NYES = NYES + 1
 166  CONTINUE
C
      FRACT = (DFLOAT(NYES)/DFLOAT(M))
 300  FORMAT(2X,' FRACTION OF TIME METHOD #1 IS CLOSER')
 303  FORMAT(2X,'                 IN  POSITION    :  ',F7.3)
 302  FORMAT(2X,'                 IN  TIME        .  ',F7.3)
      WRITE(6,300)
      WRITE(6,303) FRACT
      FRACT = (DFLOAT(NYEST)/DFLOAT(M))
      WRITE(6,302) FRACT
      WRITE(6,999)
      WRITE(6,998)
      WRITE(6,999)
C
 1222 CONTINUE
 1111 CONTINUE
C
 998  FORMAT(1X,'= = = = = = = = = = = = = = = = = = = = =')
 999  FORMAT(2X,'    ')
1000  STOP
      END
```

APPENDIX E

DOUBLE ARRAY SIMULATION COMPUTER PROGRAM

```
C        THIS FORTRAN PROGRAM SELECTS A RANDOM SAMPLE OF
C        SIZE M OF 3-DIMENSIONAL POSITIONS IN A BOX RUNNING
C        CROSSWAYS BETWEEN TWO HYDROPHONIC ARRAYS.  THE BOX
C        HAS DIMENSIONS GIVEN BY THE VECTOR BOX.  THE ARRAYS
C        ARE SEPERATED BY 7500 FEET, AND HAVE COORDINATES
C        GIVEN BY THE VECTORS A1 AND A2.  THE ARMS OF THE
C        ARRAYS ARE ALIGNED WITH THE RANGE COORDINATE SYSTEM.
C
C        THE SPEED OF SOUND PROFILE IS ASSUMED TO BE LINEAR,
C        AND IS GIVEN BY        V  =  VV(1)  + VV(2)*DEPTH  .
C
C        FOR EACH OF THE POSITIONS SELECTED, EXACT TIME VALUES
C        FOR SOUND WAVE ARRIVAL AT THE HYDROPHONES OF EACH
C        ARRAY ARE COMPUTED BY THE SUBROUTINE TCOMP.  THEN
C        RANDOM ERRORS ARE ADDED TO ALL TIMES.  THE ERROR
C        DISTRIBUTION IS NORMAL, WITH MEAN ZERO AND STANDARD
C        DEVIATION SDEV.
C
C        THE TIME VALUES ARE THEN USED TO ESTIMATE APPARENT
C        POSITIONS BY TWO DIFFERENT METHODS, USING THE
C        APPROPRIATE SUBROUTINES.  EACH METHOD PRODUCES TWO
C        DIFFERENT ESTIMATES, ONE FOR EACH ARRAY.  THE APPARENT
C        POSITIONS ARE THEN TRANSLATED TO ACTUAL POSITION
C        ESTIMATES BY THE SUBROUTINE TRACE.  THEN THE LENGTH
C        OF THE DIFFERENCE VECTOR FOR EACH METHOD IS COMPUTED.
C        THE TWO DIFFERENCE LENGTHS ARE COMPARED TO SEE WHICH
C        METHOD PRODUCES POSITION PAIRS IN CLOSEST AGREEMENT.
C
C        THIS COMPARISON IS DONE FOR ALL POSSIBLE PAIRS OF
C        THE SIX METHODS.
C
         INTEGER M,I,J,K,METH,METH1,METH2,NYES,NNOISE,NARRAY
C
         DIMENSION RP(1000),STUFF(4000),NAM1(6),NAM2(6)
C
         DOUBLE PRECISION A1(3),A2(3),T1(1000,4),T2(1000,4)
         DOUBLE PRECISION VAR(1000),NOISE,FRACT,XYZ,DEPTH
         DOUBLE PRECISION TEST(1000),P1(1000,3),P2(1000,4)
         DOUBLE PRECISION SDEV,SEED,BOX(3),PEST(1000,3),A(3)
         DOUBLE PRECISION T(1000,4),V,VV(2),DIF(1000)
C
         DATA NAM1(1),NAM1(2),NAM1(3)/'NAVY','NAVY','L.S.'/
         DATA NAM1(4),NAM1(5),NAM1(6)/'L.S.','M.L.','M.L.'/
         DATA NAM2(1),NAM2(2),NAM2(3)/'    ','CORR','    '/
         DATA NAM2(4),NAM2(5),NAM2(6)/'CORR','PLNR','SPHR'/
C
C        INITIALIZE VALUES FOR SAMPLE SIZE, RANGE, DEPTH
C        AND ERROR STANDARD DEVIATION.
C
         M = 1000
         SDEV = 3.464D-5
         DEPTH = 1300.D0
    1    FORMAT(2X,' NOISE STD. DEV.  =  ',F15.10)
         WRITE(6,1) SDEV
         WRITE(6,999)
         SEED = 9347.6D0
C
```

```
C     SET UP ARRAY POSITIONS, AND SOUND VELOCITY PROFILE
C
      A1(1) = -3750.D0
      A1(2) = 0.D0
      A1(3) = DEPTH
      A2(1) = 3750.D0
      A2(2) = 0.D0
      A2(3) = DEPTH
      VV(1) = 4840.7D0
      VV(2) = 3314.D-5
      V = VV(1)+VV(2)*DEPTH
C
 30   FORMAT(2X,'             ( SAMPLE SIZE   =  ',I5,' )')
      WRITE(6,30) M
      WRITE(6,999)
C
C     DRAW UNDERWATER BOX FOR RANDOM POSITIONS
C
      BOX(1) = 1000.D0
      BOX(2) = 5000.D0
      BOX(3) =  DEPTH
C
C     GENERATE RANDOM POSITIONS IN BOX
C
      DO 11 J = 1,2
         CALL GGUBS (SEED,M,RP)
         DO 22 I = 1,M
            XYZ = RP(I)
            P1(I,J) = (XYZ-0.5D0)*BOX(J)
 22      CONTINUE
 11   CONTINUE
      CALL GGUBS(SEED,M,RP)
      DO 33 I = 1,M
         XYZ = RP(I)
         P1(I,3) = XYZ*BOX(3)
 33   CONTINUE
C
C     COMPUTE EXACT HYDROPHONE ARRIVAL TIMES, AND ADD
C     RANDOM ERRORS TO TIME VALUES.
C
      NNOISE = M*4
      CALL TCOMP(P1,A1,M,VV,T1)
      CALL GGNML (SEED,NNOISE,STUFF)
      K = 1
      DO 44 I = 1,M
         DO 45 J = 1,4
            NOISE = STUFF(K)
            NOISE = NOISE*SDEV
            T1(I,J) = T1(I,J) + NOISE
            K = K+1
 45      CONTINUE
 44   CONTINUE
C
      CALL TCOMP(P1,A2,M,VV,T2)
      CALL GGNML (SEED,NNOISE,STUFF)
      K = 1
      DO 55 I = 1,M
         DO 56 J = 1,4
            NOISE = STUFF(K)
            NOISE = NCISE*SDEV
            T2(I,J) = T2(I,J) + NOISE
            K = K+1
 56      CONTINUE
 55   CONTINUE
C
```

```
C     SET UP LOOPS TO RUN THROUGH ALL PAIRS OF METHODS.
C
      DO 1111 METH1 = 1,5
         KMETH = METH1 + 1
      DO 1222 METH2 = KMETH,6
         IF (METH1 .EQ. METH2) GO TO 1222
C
 10   FORMAT(2X,'METHODS COMPARED ARE  1.  ',A4,2X,A4)
 20   FORMAT(2X,'                      AND  2.  ',A4,2X,A4)
      WRITE(6,10) NAM1(METH1),NAM2(METH1)
      WRITE(6,20) NAM1(METH2),NAM2(METH2)
      WRITE(6,999)
C
      METH = METH1
      NARRAY = 1
C
C     SET UP ARRAY BEING USED
C
 125  A(1)  = A1(1)
      A(2)  = A1(2)
      A(3)  = A1(3)
      DO 66 I = 1,M
         DO 67 J = 1,4
            T(I,J)  = T1(I,J)
 67      CONTINUE
 66   CONTINUE
C
C     CALL SUBROUTINES TO PERFORM ESTIMATION METHODS.

 130  IF (METH .NE.  1) GO TO 131
         CALL POSNAV  (T,V,M,PEST,TEST)
         GO TO  150
 131  IF (METH .NE.  2) GO TO 132
         CALL POSNVC  (T,V,M,PEST,TEST)
         GO TO 150
 132  IF (METH .NE.  3) GO TO 133
         CALL POSLS  (T,V,M,PEST,TEST)
         GO TO  150
 133  IF (METH .NE.  4) GO TO 134
         CALL POSLSC  (T,V,M,PEST,TEST)
         GO TO 150
 134  IF (METH .NE.  5) GO TO 135
         CALL POSMLP  (T,V,M,PEST,TEST,VAR)
         GO TO  150
 135  IF (METH .NE.  6) GO TO 136
         CALL POSMLS  (T,V,M,PEST,TEST,VAR)
         GO TO  150
 136  WRITE(6,137)  METH1,METH2
 137  FORMAT(2X,'CANT FIND METHODS ',I4,' AND/OR ',I4)
      GO TO 1000
C
 150  IF ( NARRAY .EQ. 2 )  GO TO 152
C
C     APPARENT POSITION ESTIMATES ARE IN LOCAL ARRAY
C     COORDINATES, AND MUST BE TRANSLATED TO TRACKING
C     RANGE SYSTEM COORDINATES.
C
      DO 144 I = 1,M
         PEST(I,1)  = PEST(I,1)  +  A1(1)
         PEST(I,2)  = PEST(I,2)  +  A1(2)
         PEST(I,3)  = A1(3)  - PEST(I,3)
 144  CONTINUE
C
C     CORRECT APPARENT POSITIONS BY RAY TRACING.
C
      CALL TRACE (PEST,P1,TEST,A,M,VV)
C
```

```
C     GO ON TO SECOND ARRAY
C
      NARRAY = 2
      A(1)  = A2(1)
      A(2)  = A2(2)
      A(3)  = A2(3)
      DO 77 I = 1,M
         DO 78 J = 1,4
            T(I,J)  = T2(I,J)
 78      CONTINUE
 77   CONTINUE
      GO TO 130
C
C     TRANSLATE 2ND ARRAY APPARENT POSITIONS TO RANGE
C     COORDINATE SYSTEM, AND THEN RAY TRACE.
C
 152  DO 145 I = 1,M
         PEST(I,1)  = PEST(I,1)  + A2(1)
         PEST(I,2)  = PEST(I,2)  + A2(2)
         PEST(I,3)  = A2(3)  - PEST(I,3)
 145  CONTINUE
C
      CALL TRACE (PEST,P2,TEST,A,M,VV)
C
C     COMPUTE DIFFERENCE VECTORS FOR 1ST METHOD
C
      IF ( METH .NE. METH1 ) GO TO 160
         DO 155 I = 1,M
           DIF(I) = 0.D0
           DO 156 J = 1,3
              DIF(I)  = DIF(I)+(P1(I,J)-P2(I,J))**2
 156       CONTINUE
 155     CONTINUE
C
C     GO ON TO SECOND METHOD
C
         METH = METH2
         NARRAY = 1
         GO TO 125
C
C     COMPUTE DIFFERENCE VECTORS FOR 2ND METHOD AND
C     COMPARE TO DIFFERENCES FOR 1ST METHOD.
C
 160  NYES = 0
      DO 166  I = 1,M
         DO 167 J = 1,3
            DIF(I)  = DIF(I)-(P1(I,J)-P2(I,J))**2
 167     CONTINUE
         IF ( DIF(I)  .GT. 0.D0 )  GO TO 166
            NYES = NYES + 1
 166  CONTINUE
C
      WRITE(6,999)
      FRACT =  (DFLOAT(NYES) /DFLOAT(M))
 300  FORMAT(2X,' FRACTION  OF TIME METHOD #1 ')
 310  FORMAT(2X,'          IS BETTER IS     : ',F7.3)
      WRITE(6,300)
      WRITE(6,310)  FRACT
      WRITE(6,999)
      WRITE(6,998)
      WRITE(6,999)
C
 1222 CONTINUE
 1111 CONTINUE
C
 998  FORMAT(1X,'= = = = = = = = = = = = = = = = = = = = = = ')
 999  FORMAT(2X,'   ')
 1000 STOP
```

APPENDIX F

COMPUTER SUBROUTINES FOR TIME CALCULATION AND RAYTRACING

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE TCOMP (P,A,M,VV,T)
C
C     THIS FORTRAN SUBROUTINE COMPUTES THE EXACT TIME
C     REQUIRED FOR A SOUND WAVE TO TRAVEL FROM ITS
C     SOURCE (VECTOR P) TO THE FOUR HYDROPHONES ON AN
C     ARRAY WHOSE ACOUSTIC CENTER IS SPECIFIED BY
C     VECTOR A.
C
C     THE METHODOLOGY USED IS SET FORTH IN APPENDIX A
C     OF THE THESIS.  THE BASIC ASSUMPTION IS ONE OF
C     A LINEAR VELOCITY PROFILE, WHOSE COEFFICIENTS ARE
C     GIVEN BY THE VECTOR VV.
C
      INTEGER M,I,J
      DOUBLE PRECISION  P(1000,3),A(3),VV(2),T(1000,4)
      DOUBLE PRECISION  AA(4,3),P1,P2,R,C1,A2,C2
C
C     SET UP COORDINATES OF FOUR HYDROPHONES ON THE ARRAY
C
      DO 11 I = 1,4
         DO 22 J = 1,3
            AA(I,J) = -15.D0 + A(J)
 22      CONTINUE
         AA(I,3) = 15.D0 + A(3)
 11   CONTINUE
      AA(1,1) = 15.D0 + A(1)
      AA(2,2) = 15.D0 + A(2)
      AA(3,3) = -15.D0 + A(3)
C
C     CALCULATE QUANTITIES C1, C2, R, AND THE TIMES T(*,4)
C
      C2 = -1.D0*(VV(1)/VV(2))
C
C     LOOP THROUGH THE M SOURCE LOCATIONS
      DO 33 I = 1,M
         P2 = P(I,3)
C        LOOP THROUGH THE FOUR HYDROPHONES FOR EACH SOURCE
         DO 44 J = 1,4
            P1 = DSQRT((P(I,1)-AA(J,1))**2
     +                + (P(I,2)-AA(J,2))**2)
            C1 = P1**2+P2**2-AA(J,3)**2
     +                +2.D0*C2*(AA(J,3)-P2)
            C1 = C1/(2.D0*P1)
            R = DSQRT(C1**2+(AA(J,3)-C2)**2)
            T(I,J) = ((AA(J,3)-C2)/(P2-C2))
     +                        *((R-C1+P1)/(R-C1))
            T(I,J) = (DLOG(T(I,J)))/VV(2)
 44      CONTINUE
 33   CONTINUE
C
      RETURN
      END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE  TRACE (P,PT,T,A,M,VV)
C
C     THIS FORTRAN SUBROUTINE TAKES AN APPARENT POSITION
C     ESTIMATE P AND CONVERTS IT TO AN ACTUAL POSITION
C     ESTIMATE PT BY RAY TRACING.  THE RAY TRACING ASSUMES
C     A LINEAR VELOCITY PROFILE, WITH COEFFICIENTS IN THE
C     VECTOR VV.  OTHER INPUTS ARE THE ARRAY LOCATION
C     VECTOR A AND THE RAY TRANSIT TIME TO THE ACOUSTIC
C     CENTER, T.  THE RAY TRACING LAYERS ARE DEL FEET THICK
C     WITH THE FIRST (BOTTOM) LAYER BEING THE FIRST DEL
C     FEET IMMEDIATELY ABOVE THE ARRAY.
C
C
      INTEGER  M, I
      DOUBLE PRECISICN T(1000),P(1000,3),PT(1000,3),A(3)
      DOUBLE PRECISION DEL,H,SA,CA,DEPTH,V,V1,VV(2)
      DOUBLE PRECISICN DS,DT,DZ,DH
C
      DEL = 25.D0
C
C     LOOP THROUGH THE M APPARENT POSITIONS
C
      DO 11 I = 1,M
C        INITIALIZE INCREMENTAL VALUES FOR EACH POSITION
         H = DSQRT((P(I,1)-A(1))**2+(P(I,2)-A(2))**2)
         CA = H/DSQRT(H**2+(P(I,3)-A(3))**2)
         DT = 0.D0
         DZ = 0.D0
         DH = 0.D0
         DEPTH = A(3) - DEL/2.D0
         V = VV(1)+VV(2)*DEPTH
C
C     INNER LOOP : PROCESS DATA UPWARDS, LAYER BY LAYER,
C        UNTIL RAY TRANSIT TIME IS EXHAUSTED
C
 10      SA = DSQRT(1.D0-CA**2)
         DZ = DZ + DEL
         DS = DEL/SA
         DT = DT + DS/V
         DH = DH + DS*CA
C
         IF ( DT .GT. T(I) ) GO TO 20
         DEPTH = DEPTH - DEL
         V1 = VV(1) + VV(2)*DEPTH
         CA = CA*(V1/V)
         V = V1
         GO TO 10
C
C     ADJUST FOR OVERSHOOTING IN LAST LAYER
C
 20      DS = V*(DT-T(I))
         DH = DH - CA*DS
C
C     ADJUST APPARENT POSITION TO GET ACTUAL POSITION
C
         PT(I,1) = (P(I,1)-A(1))*(DH/H)+A(1)
         PT(I,2) = (P(I,2)-A(2))*(DH/H)+A(2)
         PT(I,3) = A(3) - (DZ-SA*DS)
C
 11   CONTINUE
      RETURN
      END
```

APPENDIX G

COMPUTER SUBROUTINES FOR POSITION ESTIMATION METHODS

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE  POSNAV (T,V,M,P,NT)
C
      INTEGER  M, I
C
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE ORIGINAL
C     UNADJUSTED NAVY APPARENT POSITION ESTIMATION METHOD.
C
C     INPUT ARE THE HYDROPHONE TIMES T FOR M SOUND SOURCE
C     POSITIONS, AND VELOCITY V AT THE ARRAY.
C
C     OUTPUT ARE M APPARENT POSITION ESTIMATES P ALONG WITH
C     THE CORRESPONDING M RAY TRANSIT TIMES NT.  ALL THE
C     POSITIONS ARE REFERENCED TO THE ACOUSTIC CENTER, WITH
C     THE Z COMPONENT BEING MEASURED UPWARDS FROM THE ARRAY.
C
      DOUBLE PRECISION T(1000,4),P(1000,3),NT(1000)
      DOUBLE PRECISION V,TC,D,DISC,NUMER,DENOM
C
      D = 30.D0
      DO 11 I = 1,M
         NUMER = 0.D0
         DENOM = 0.D0
         TC = T(I,4)
         DO 22 J = 1,3
            P(I,J) = V*V*(TC*TC-T(I,J)**2)/(D*2.D0)
            NUMER = NUMER + P(I,J)**2
            DENOM = DENOM + (P(I,J) + 15.D0)**2
 22      CONTINUE
         NT(I) = TC*DSQRT(NUMER/DENOM)
 11   CONTINUE
      RETURN
      END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE POSNVC (T,V,M,P,NT)
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE ADJUSTED NAVY
C     METHOD NAVY_A FOR ESTIMATING APPARENT POSITIONS.
C
C     INPUT ARE THE HYDROPHONES TIMES T FOR M SOUND SOURCE
C     LOCATIONS, AND THE VELOCITY V AT THE ARRAY.
C
C     OUTPUT ARE THE M APPARENT POSITION ESTIMATES P, AND
C     THE CORRESPONDING M RAY TRANSIT TIMES NT.  ALL
C     POSITIONS ARE REFERENCED TO THE ACOUSTIC CENTER OF
C     THE ARRAY, WITH THE Z COMPONENT MEASURE UPWARDS
C     FROM THE ARRAY.
C
      INTEGER M,I
      DOUBLE PRECISION  T(1000,4),P(1000,3),NT(1000)
      DOUBLE PRECISION  V,D,NUMER,DENOM,TC,DCC
C
      D = 30.D0
      DO 11 I = 1,M
         DCC = 0.D0
         TC = T(I,4)
         DO 22 J = 1,3
            P(I,J) = 15.D0 + (V*V*(TC*TC-T(I,J)**2)/(D*2.D0))
            DCC = DCC + P(I,J)**2
 22      CONTINUE
         NUMER = 0.D0
         DENOM = 0.D0
         DO 44 J = 1,3
            P(I,J) = (P(I,J)*V*TC)/DSQRT(DCC)
            DENOM = DENOM + P(I,J)**2
            P(I,J) = P(I,J) - 15.D0
            NUMER = NUMER + P(I,J)**2
 44      CONTINUE
         NT(I) = TC*DSQRT(NUMER/DENOM)
 11   CONTINUE
      RETURN
      END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE POSLS (T,V,M,P,LST)
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE LEAST SQUARES
C     PLANAR METHOD L.S.
C
C     INPUT ARE THE HYDROPHONE TIMES T FOR M SOUND SOURCE
C     POSITIONS, AND THE VELOCITY V AT THE ARRAY.
C
C     INPUTS AND OUTPUTS ARE THE SAME AS FOR THE
C     SUBROUTINE POSNAV.
C
      INTEGER M,I,J
      DOUBLE PRECISION T(1000,4),P(1000,3),LST(1)00)
      DOUBLE PRECISICN V,DISC,TC
C
      DO 11 I = 1,M
        DISC = 0.D0
        DO 22 J = 1,3
           P(I,J) = T(I,4) -T(I,J)
           DISC = DISC + P(I,J)**2
 22     CONTINUE
        LST(I) = (T(I,1)+T(I,2)+T(I,3)-T(I,4))/2.D0
        DO 33 J = 1,3
           P(I,J) = (V*LST(I)*P(I,J))/DSQRT(DISC)
 33     CONTINUE
 11   CONTINUE
C
      RETURN
      END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE POSISC (T,V,M,P,LST)
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE BIAS ADJUSTED
C     LEAST SQUARES PLANAR METHOD L.S.C. FOR ESTIMATING
C     APPARENT POSITIONS.
C
C     INPUTS AND OUTPUTS ARE THE SAME AS FOR THE
C     SUBROUTINE POSIS.
C
      INTEGER I,M,J,K,L
      DOUBLE PRECISION T(1000,4),P(1000,3),LST(1)00)
      DOUBLE PRECISION V,A(4,3),D(4),DISC,TD,R1SQR,R2SQR,E(3)
C
C     SET UP HYDROPHONE POSITIONS
      DO 44 J =1,3
         DO 55 I = 1,4
            A(I,J) = -15.D0
 55      CONTINUE
         A(J,J) = 15.D0
 44   CONTINUE
C
C     CALCULATE ORIGINAL L.S. SOLUTION
      DO 11 I = 1,M
        DISC = 0.D0
        DO 22 J = 1,3
           P(I,J) = T(I,4) -T(I,J)
           DISC = DISC + P(I,J)**2
 22     CONTINUE
        LST(I) = (T(I,1)+T(I,2)+T(I,3)-T(I,4))/2.D0
        DO 33 J = 1,3
           P(I,J) = (V*LST(I)*P(I,J))/DSQRT(DISC)
 33     CONTINUE
        R1SQR = P(I,1)**2+P(I,2)**2+P(I,3)**2
        D(4) = 0.D0
        DO 89 K = 1,3
           D(4) = D(4) + (P(I,K) - A(4,K))**2
 89     CONTINUE
        D(4) = DSQRT(D(4))
        DISC = 0.D0
        TD = 0.D0
        DO 77 J = 1,3
           D(J) = 0.D0
           DO 88 K = 1,3
              D(J) = D(J) + (P(I,K) - A(J,K))**2
 88        CONTINUE
           D(J) = DSQRT(D(J))
           DISC = DISC + (D(4)-D(J))**2
           TD = TD + D(J)
 77     CONTINUE
C
C       CALCULATE BIAS VECTOR E
        DISC = DSQRT(DISC)
        DO 66 J = 1,3
           E(J) = ((TD-D(4))*(D(4)-D(J))/(DISC*2.D0))-P(I,J)
 66     CONTINUE
        R2SQR = 0.D0
        DO 99 J =1,3
           P(I,J) = P(I,J) - E(J)
           R2SQR = R2SQR + P(I,J)**2
 99     CONTINUE
C
C       ADJUST ORIGINAL RAY TRANSIT TIME
        LST(I) = LST(I)*(DSQRT(R2SQR/R1SQR))
 11   CONTINUE
      RETURN
      END
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE  POSMLP (T,V,M,P,MT,VAR)
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE MAXIMUM
C     LIKELIHOOD PLANAR METHOD M.L.P. FOR ESTIMATING
C     APPARENT POSITIONS.
C
C     INPUTS ARE THE HYDROPHONE TIMES T FOR M SOURCE
C     POSITIONS, AND THE VELOCITY OF SOUND V AT THE ARRAY.
C
C     OUTPUTS ARE THE APPARENT POSITION ESTIMATES P AND
C     RAY TRANSIT TIMES MT FOR EACH OF THE M POSITIONS.
C     ALSO OUTPUT IS A VECTOR VAR OF M ESTIMATES OF THE
C     TIMING ERROR VARIANCE.  POSITIONS ARE ALL REFERENCED
C     TO THE ACOUSTIC CENTER OF THE ARRAY, WITH THE Z
C     COMPONENT MEASURED UPWARDS FROM THE ARRAY.
C
      INTEGER  M, I, J
      DOUBLE PRECISION T(1000,4),P(1000,3),MT(1000)
      DOUBLE PRECISION V,TC,D,DIFF,NUMER,DENOM,TOL,C(3)
      DOUBLE PRECISION VAR(1000),CO(3),X(3)
C
      D = 30.D0
      TOL = 1.D-4
C
C     INITIALIZE THE DIRECTION COSINE ESTIMATE BY
C     USING THE LEAST SQUARES SOLUTION.
C
C     LOOP THROUGH THE M POSITIONS
C
      DO 11 I = 1,M
         TC = T(I,4)
         DENOM =  (TC-T(I,1))**2+(TC-T(I,2))**2
     +                           +(TC-T(I,3))**2
         DO 12 J = 1,3
            C(J) =  (TC - T(I,J))/DSQRT(DENOM)
 12      CONTINUE
C
         ITER = 0
 22      DENOM = 0.D0
         ITER = ITER + 1
         TC = 0.D0
C
C        USE ITERATION FORMULAE TO DEVELOPE COSINES
C        AND TIME VALUES.
C
         DO 33 J = 1,3
            CO(J) = C(J)
            DENOM = DENOM + T(I,J)*C(J)
            TC = TC + T(I,J)
 33      CONTINUE
         TC =  ((TC+T(I,4))+D*(C(1)+C(2)+C(3))/V)/4.D0
         DENOM = DENOM - TC*(C(1)+C(2)+C(3))
         DO 66 J = 1,3
            C(J) =  (T(I,J)-TC)/DENOM
            X(J) = DABS(CO(J)-C(J))
 66      CONTINUE
C
C        CHECK ITERATION TOLERANCES
C
         DIFF = DMAX1(X(1),X(2),X(3))
         IF ( ITER .LT. 100) GO TO 70
 60         FORMAT (2X,'ITERATIONS EXCEED ',I6)
 61         FORMAT (2X,'IN POSITION NO.   ',I6)
            WRITE(6,60) ITER
            WRITE(6,61) I
            GO TO 11
```

```
 70       IF ( TOL .LT. DIFF ) GO TO 22
C
          NUMER = 0.D0
          DENOM = 0.D0
C
C         CONVERT COSINES TO POSITIONS, AND TRANSLATE TO
C         ACOUSTIC CENTER REFERENCED COORDINATES.
C
          DO 44  J = 1,3
             P(I,J)  =  V*C(J) *TC
             DENOM  =  DENOM  +  P(I,J)**2
             P(I,J)  =  P(I,J)  -15.D0
             NUMER  =  NUMER  +  P(I,J)**2
 44       CONTINUE
          MT(I)  =  TC*DSQRT(NUMER/DENOM)
C
C         ESTIMATE VARIANCE
C
          VAR(I)  =  0.D0
          DO 55  J = 1,3
             VAR(I)  =  VAR(I)  +  (T(I,J)-TC+D*C(J)/7)**2
 55       CONTINUE
          VAR(I)  =  VAR(I)/4.D0
 11    CONTINUE
       RETURN
       END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE  POSMLS (TIME,V,M,P,MLT,VARML)
C
C     THIS FORTRAN SUBROUTINE IMPLEMENTS THE MAXIMUM
C     LIKELIHOOD SPHERICAL METHOD M.L.S. FOR ESTIMATING
C     APPARENT POSITIONS.
C
C     INPUTS AND OUTPUTS ARE THE SAME AS FOR THE
C     SUBROUTINE POSMIP.
C
      INTEGER  M, I, J, IER, II, ITER1, ITER2
      DOUBLE PRECISION TIME(1000,4),P(1000,3),VARML(1000)
      DOUBLE PRECISION MLT(1000),SDEV(1000),K(3), DSDC(3)
      DOUBLE PRECISION V,R,D,TOL1,TOL2,EPS,L,S,DIFF
      DOUBLE PRECISION T(3),TC,TAU,TK(3),VK(3),C(3),C1(3)
      DOUBLE PRECISION X(3),DENOM,NUMER,DLDC(3),DLDT,DSDT
      DOUBLE PRECISION GP(4,4),GPI(4,4),WKAREA(28),GC(4)
      DOUBLE PRECISION GT,X1(4),X2(4),A(4),B(4),T1
      DOUBLE PRECISION F1,F2
C
C   SET GLOBAL VALUES
C
      R = (3.D0-DSQRT(5.D0))/2.D0
      D = 30.D0
      TOL1 = 1.D-5
      TOL2 = 1.D-5
C
C   START OUTERMOST LOOP (ONE FOR EACH SOURCE POSITION)
C
      DO 5 II = 1,M
         DENOM = 0.D0
         TAU = TIME(II,4)
         EPS = 1.D-3
         TC = TAU
         DO 111 J = 1,3
            T(J) = TIME(II,J)
            C(J) = ((V**2)*(TC**2-T(J)**2)/(D*2.D))+D/2.D0
            DENOM = DENOM + C(J)**2
 111     CONTINUE
C
         DO 122 J = 1,3
            C(J) = C(J)/DSQRT(DENOM)
 122     CONTINUE
C
C   START MIDDLE LOOP  (CREATE DERIVATIVE MATRIX GP(,) )
C
         ITER1 = 0
 10      T1 = TAU
         ITER1 = ITER1 + 1
         IF (ITER1 .LT. 50 ) GO TO 300
 310        FORMAT(2X,'EXCESSIVE NEWTON ITERS.')
 311        FORMAT(2X,'              IN POSITION   ',I6)
            WRITE(6,310)
            WRITE(6,311) II
            GO TO 5
 300     L = 0.D0
         S = 0.D0
         DLDT = 0.D0
         DSDT = 0.D0
C
         DO 11 I = 1,3
            C1(I) = C(I)
            K(I) = TAU**2+(D/V)**2-(2.D0*D*TAU*C(I))/V
            VK(I) = V*K(I)*DSQRT(K(I))
            TK(I) = T(I)-DSQRT(K(I))
            DLDC(I) = ((V*K(I)*TK(I))+C(I)*T(I)*D*TAU)/VK(I)
            DSDC(I) = (T(I)*D*TAU)/VK(I)
            DLDT = DLDT+(C(I)*T(I)*(D*C(I)-V*TAU))/VK(I)
```

```
          DSDT  =  DSDT+T(I)*(D*C(I)-V*TAU)/VK(I)
          L =  L+(C(I)*TK(I))/DSQRT(K(I))
          S =  S+TK(I)/DSQRT(K(I))
 11     CONTINUE
C
        DO 22 I  =  1,3
          DO 33 J  =  1,3
             GP(I,J)=(TK(I)*DLDC(J))/(L*L*DSQRT(K(I)))
 33       CONTINUE
          GP(4,I)=DSDC(I)*(V*TC-D*L)
          GP(4,I)=(GP(4,I)-D*DLDC(I)*(1.D0-S))
     *                                         /(V*(1.D0-S)**2)
          GP(I,I)=(I*T(I)*D*TAU)-DLDC(I)*K(I)*V*TK(I)
          GP(I,I)  =  1.D0  -  (GP(I,I)/(VK(I)*L*L))
          GP(I,4)=(T(I)*L*(V*TAU-D*C(I))
     *                                +V*K(I)*C K(I)*DLDT
          GP(I,4)  =  GP(I,4)  /  (VK(I)*L*L)
 22     CONTINUE
        GP(4,4)=1.D0-((DSDT*(V*TC-D*L))-D*DLDT*(1.D0-S))
     *                                         /(V*(1.D0-S)**2)
C
C   INVERT DERIVATIVE MATRIX
C
        CALL GAUSS3(4,0,GP,GPI,IER,4)
C
C    CALCULATE INITIAL NEWTON SEARCH SOLUTION
C
 40     DO 44 I  =  1,3
          GC(I)  =  C(I)-TK(I)/(DSQRT(K(I))*L)
 44     CONTINUE
          GC(4)  =  TAU-(V*TC-D*L)/(V*(1.D0-S))
        B(4)  =  TAU
        DO 55 I  =  1,3
          B(4)  =  B(4)-GPI(4,I)*GC(I)
          B(I)  =  C(I)
          DO 56 J  =  1,4
             B(I)  =  B(I)-GPI(I,J)*GC(J)
 56       CONTINUE
 55     CONTINUE
        B(4)  =  B(4)  -  GPI(4,4)*GC(4)
C
C    PREPARE FOR GOLDEN SECTION SEARCH
C
        EPS  =  DMAX1((1.D-6),(EPS/1.D1))
        DO 66 I  =  1,3
          A(I)  =  C(I)
          X1(I)  =  A(I)+R*(B(I)-A(I))
          C(I)  =  X1(I)
 66     CONTINUE
        A(4)  =  TAU
        X1(4)  =  A(4)+R*(B(4)-A(4))
        TAU  =  X1(4)
C
C    START INNERMOST LOOP ( COMMENCE GOLDEN SECTION SEARCH
C          TO IMPROVE THE INITIAL NEWTON SOLUTION )
C
        ITER2  =  0
 70     CALL OBJFCN  (F1,L,K,TK,S,X1,D,V,T,TC)
        ITER2  =  ITER2  +  1
        IF  (ITER2 .LT. 50 )  GO TO 400
 410      FORMAT(2X,'EXCESSIVE G.S.S. ITERS., POS #',I6)
          WRITE(6,410)  II
          GO TO 5
 400    DIFF  =  0.D0
        DO 77 I  =  1,3
          DIFF  =  DIFF  +  (A(I)-B(I))**2
          X2(I)  =  A(I)  +  B(I)  -  X1(I)
          C(I)  =  X2(I)
 77     CONTINUE
```

```
      X2(4)  =  A(4)  + B(4)  -  X1(4)
      TAU = X2(4)
      DIFF =  DSQRT(DIFF + (A(4)-B(4))**2)
C
C   CHECK TOLERANCES - STOP G.S.S. IF TIGHT ENOJGH
C
      IF ( EPS .GT. DIFF ) GO TO 100
C
      CALL OBJFCN(F2,L,K,TK,S,X2,D,V,T,TC)
C
C   CHOOSE IMPROVING DIRECTION
C
      IF ( F1 .LT. F2 ) GO TO 90
      DO 88 I = 1,3
         A(I)  = X1(I)
         X1(I)  =  X2(I)
         C(I)  = X1(I)
 88   CONTINUE
      A(4)  = X1(4)
      X1(4)  =  X2(4)
      GO TO 70
C
 90   DO 99 I = 1,3
         B(I)  = X2(I)
         X1(I)  =  A(I)+B(I)-X1(I)
         C(I)  = X1(I)
 99   CONTINUE
      B(4)  = X2(4)
      X1(4)  =  A(4)+B(4)-X1(4)
      GO TO 70
C
C    CHECK TOLERANCES FOR NEWTON SEARCH ITERATIONS,
C          AND PREP FOR NEXT NEWTON ITERATION IF NECESSARY
C
 100  DO 144 I = 1,3
         X(I)  = DABS(C(I)-C1(I))
 144  CONTINUE
      DIFF = DMAX1(X(1),X(2),X(3))
      IF ( TOL1 .LT. DIFF ) GO TO 10
      DIFF = DABS(TAU-T1)
      IF ( TOL2 .LT. DIFF ) GO TO 10
C
C   DONE WITH  II-TH SET OF TIMES AND POSITIONS.   MAKE
C   ESTIMATES, AND GO ON TO NEXT POSITION TO BE ESTIMATED
C
      VARML(II) = 0.D0
      DENOM = 0.D0
      NUMER = 0.D0
      DO 155 J = 1,3
         VARML(II) = VARML(II)  + TK(J)*TK(J)
         P(II,J)  =  V*TAU*C(J)
         DENOM = DENOM  + P(II,J)**2
         P(II,J)  =  V*TAU*C(J)-D/2.D0
         NUMER = NUMER  + P(II,J)**2
 155  CONTINUE
      VARML(II)  =  (VARML(II)+(TC-TAU)**2)/4.D0
      SDEV(II)  =  DSQRT(VARML(II))
      MLT(II)  = TAU*DSQRT(NUMER/DENOM)
 5   CONTINUE
C     WRITE(7,900)  (SDEV(I),I = 1,M)
C900  FORMAT(5E15.5)
     RETURN
     END
C
```

```
C SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
      SUBROUTINE OBJFCN (F,L,K,TK,S,X,D,V,T,TC)
C
C     THIS FORTRAN SUBROUTINE IS CALLED BY THE SUBROUTINE
C     POSMLS.  IT CALCULATES NEW VALUES FOR SEVERAL
C     VARIABLES AS WELL AS A FUNCTIONAL VALUE WHICH IS THE
C     DECISION FACTOR DETERMINING THE APPROPRIATE
C     IMPROVING DIRECTION FOR THE GOLDEN SECTION SEARCH.
C
      INTEGER I
      DOUBLE PRECISION F,L,K(3),TK(3),S,X(4),D,V,TC,T(3)
C
      S = 0.D0
      L = 0.D0
      DO 11 I = 1,3
         K(I) = X(4)**2+(D/V)**2-((2.D0*D*X(4)*X(I))/V)
         TK(I) = T(I)-DSQRT(K(I))
         S = S + TK(I)/DSQRT(K(I))
         L = L + (X(I)*TK(I))/DSQRT(K(I))
 11   CONTINUE
      F = 0.D0
      DO 22 I = 1,3
         F = F+(X(I)-TK(I)/(DSQRT(K(I))*L))**2
 22   CONTINUE
      F = F+(X(4)-(V*TC-D*L)/(V*(1.D0-S)))**2
      RETURN
      END
```

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Thesis
M277227 Main
c.1 Alternative models for calculation of evaluation angles and ray transit times for ray tracing of hydrophonic tracking data.